



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year
SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE (MATHEMATICS)

SMA 361-THEORY OF ESTIMATION

DATE: 25/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Explain the meaning of the following terms as applied in theory of estimation
- Population
 - An estimator (4 marks)
- b) Let $x_1, x_2, \dots, x_n$ be a random sample drawn from a population with mean μ (unknown). Show that if $y = \sum_{i=1}^n a_i x_i$ is unbiased estimator for μ if $\sum_{i=1}^n a_i = 1$ (6 marks)
- c) Let $x_1, x_2, \dots, x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$, $x=0,1,\dots$. Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ (4 marks)
- d) Let $x_1, x_2, \dots, x_n$ be a random sample from a population with mean μ and variance δ^2 .

Consider the following estimators for μ .

$$t_1 = \frac{1}{2}(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \dots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
 - Determine the efficiency of t_3 relative to t_2 (5 marks)
- e) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a normally distributed population with mean μ (unknown) and known variance δ^2 . Determine the maximum likelihood estimator (MLE) of μ . (6 marks)

QUESTION TWO (20 MARKS)

- a) Use the following data to fit the regression line $y = \alpha + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8 marks)

- b) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a population whose pdf is given by
 $f(x, \theta) = \theta e^{-\theta x}, x > 0,$

Determine the Cramer-Rao lower bound for θ .

(12 marks)

QUESTION THREE (20 MARKS)

- a) Let $x_1, x_2, \dots, x_n$ be a random sample from $x \sim N(\mu_0, \delta^2)$ where μ, δ^2 are both unknown. Determine a joint sufficient statistic of μ , and δ^2 (8 marks)
- b) A random sample of size n is drawn from a normal population with $(0, \delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (12 marks)

QUESTION FOUR (20 MARKS)

- a) Let $x_1, x_2, \dots, x_n$ denote a random sample from a pdf

$$f(x) = \begin{cases} (\theta + 1)x^\theta, & 0 < x < 1, \theta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Obtain the moment estimator of θ

(8 marks)

- b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased.

(12 marks)

QUESTION FIVE (20 MARKS)

- a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (8 marks)
- b) If $x_1, x_2, \dots, x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)