# MACHAKOS UNIVERSITY

**University Examinations 2016/2017**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE IN**

**BACHELOR OF EDUCATION (SCIENCE)**

**BACHELOR OF EDUCATION ARTS**

**SMA 407: MEASURE THEORY**

**DATE: 2/6/2017** **TIME: 2:00 – 4:00 PM**

**INSTRUCTION**

**Answer ALL the questions in Section A and ANY TWO Questions in Section B**

**SECTION(COMPULSORY)**

**QUESTION ONE (30 MARKS)**

a) Define a measure? (3 marks)

b) Prove that if $\mu$ is a measure defined on a $\sigma -$algebra $\mathfrak{X}$. Then $\mu$ is monotonic, that is if $E \complement F$, then $\mu(E) \leq \mu(F)$. Furthermore if $\mu(E) \leq \infty$, then $\mu(F-E) = \mu(F) - \mu(E)$.
. (5 marks)

c) Prove that $\mu^*(\{x\}) = 0$ for all $x \in \mathbb{R}$ (4 marks)

d) Show that if $\mu^*(E) = 0$, then E is L-measurable. (5 marks)

e) Prove that the space $(\mathbb{R}, \mathcal{M}, \mu)$, where $\mu$ is the lebesque measure is complete. (5 marks)

f) Prove that if $f, g\colon \rightarrow \mathbb{R}$ are two $\mathfrak{X} -$measurable functions and $c$ be a real number then the function $cf$ is $\mathfrak{X} -$measurable (5 marks)

g) Prove that if $\varphi$ and $\rho$ are simple functions in $M^+(X, \mathfrak{X})$ and $c \geq 0$, then $\int c\varphi du = c \int \varphi du$ (3 marks)

**SECTION B: ANSWER ANY OTHER TWO QUESTIONS**

**QUESTION TWO (20 MARKS)**

a) Prove that if $f, g\colon \to \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and $c$ be a real number then the function

i) $f^2$ (5 marks)

ii) $|f|$ are $\mathfrak{X}$ −measurable. (5 marks)

b) Let $(f_n)$ be a sequence of $\mathfrak{X}$ −measurable functions $f_n\colon X \to \mathbb{R}$ , then the functions $f_1 \cup f_2 \cup f_{3\cup} \dots \dots \dots \cup f_n$ and $f_1 \cap f_2 \cap f_3 \cap \dots \dots \dots \cap f_n$ is $\mathfrak{X}$ −measurable. (10 marks)

**QUESTION THREE (20 MARKS)**

a) State and prove monotone convergence theorem. (10 marks)

b) Let $(X, \mathfrak{X})$ be a measurable space. Then if $f, g \in M^+(X, \mathfrak{X})$, then

i) $f + g \in M^+(X, \mathfrak{X})$ (6 marks)

ii) $\int cf du = c \int f du$ (4 marks)

**QUESTION FOUR (20 MARKS**

a) Prove that if $f$ and $g$ both belong to $M^+$ and $f \leq g$, then $\int f du \leq \int g du$ (6 marks)

b) Prove that if $f \in M^+$ and if $E, F \in \mathfrak{X}$, with $E \complement F$ then $\int_E f du \leq \int_F f du$ (7 marks)

c) State and prove Fatous lemma. (7 marks)

**QUESTION FIVE (20 MARKS)**

a) Define $\sigma$ −algebra (5 marks)

b) Prove that if $A \complement B$ then $\mu^*(A) \leq \mu^*(B)$, $A, B \in \mathbb{R}$ (5 marks)

c) Prove that $\mu^*(\emptyset) = 0$ (3 marks)

d) Prove that lebesque outer measure $\mu^*$ is countably sub additive. (7 marks)