MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF ARTS

SMA 434: GAS DYNAMICS

DATE: 6/5/2019 **TIME: 8:30 -10:30 AM**

INSTRUCTIONS:

Answer question **one** (**Compulsory**) and any other **two** questions.

QUESTION ONE (COMPULSORY)(30 MARKS)

a) Define the following terms;

i) Isentropic process (2 marks)

ii) Mach cone (2 marks)

iii) Oblique shock wave (2 marks)

b) Compute the density of air at 7200 Nm^{-2}, 380 K.

Take the gas constant $R=53.3Jkg^{-1}K^{-1}$ (2 marks)

c) Considering a closed system, that executes a complete cycle, state the first law of thermodynamics (2 marks)

d) Consider a gas flowing through a stream tube. Given that at one section of the tube, the flow has a pressure of 200 Nm^{-2}, a temperature of 300 K and is moving with a velocity of 250 ms^{-1}, determine the temperature at another section of the tube where the pressure reduces to 40 Nm^{-2} and the speed increases to 1100 ms^{-1}. *Take the specific heat of the gas at constant pressure to be* $c_p = 2295\ Jkg^{-1}K^{-1}$. (4 marks)

e) Hydrogen gas has a static temperature of 300 K and stagnation temperature of 540 K. determine its Mach number and hence classify the disturbance as either subsonic or supersonic. *Take the ratio of specific heats to be* $\gamma = 1.41$. (3 marks)

f) Consider a flow between a slot separated by two plates; the lower at $y = 0$ is stationery and the one at $y = h$, moving with a velocity U. Given that the velocity is only in the x direction and only a function of y, the flow has no imposed pressure gradient or body force, the flow has no acceleration, assuming constant viscosity and using the linear momentum principle, show that the flow is linear. (5 marks)

g) Consider air at a temperature of 640 K flowing through a converging –diverging nozzle which has a cross sectional area of 1 cm^2. Given that the initial mass flow rate is 0.04125 kgs^{-1}, the initial and exit pressures are 210 kPa and 191.5 kPa. Determine;

i) Mass flow rate at the exit, (3 marks)

ii) Exit velocity. (5 marks)

Take $\gamma = 1.4$, $R = 287\ Jkg^{-1}K^{-1}$ *and* $c_p = 1004.5\ Jkg^{-1}K^{-1}$.

QUESTION TWO (20 MARKS)

a) Consider a flow between a slot separated by two plates; the lower at $y = 0$ is stationery and the one at $y = h$, moving with a velocity U. Given that the velocity is only in the x direction and only a function of y, the flow is driven by a pressure difference in the x direction such that at $x = 0, P = P_o$ and at $x = l, P = P_1$. If there is no body force, the flow has no acceleration, assuming constant viscosity and using the linear momentum principle, determine the velocity profile parameterized by P_o, P, h, U and μ. (10 marks)

b) Consider a calorically perfect ideal gas. Given that $PV = RT$ and $e = c_v T + e_0$ show that for a gas undergoing an isentropic process between two planes, the internal energy change is given by; (7 marks)

$$e_2 - e_1 = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

c) Given that the speed of sound in a general material $c(T,\rho) = \sqrt{\left(\frac{dp}{d\rho}\right)_T + \frac{T}{c_v\rho^2}\left[\left(\frac{dP}{dT}\right)_\rho\right]^2}$

show that for an ideal gas the speed is only a function of temperature given by

$c(T) = \sqrt{\gamma RT}$ (3 marks)

QUESTION THREE (20 MARKS)

a) Consider a calorically perfect ideal gas flowing through a duct of length 150m having a uniform cross section of area 0.03 m^2. Given that at one end of the duct, the gas has a pressure of 150 kPa, a temperature of 300 K and is flowing at a velocity of 12 ms^{-1}. If the gas is isobarically heated with a constant heat flux along the entire surface of the duct such that at the other end it has a pressure of 150 kPa, a temperature of 500K, determine;

  i) The mass flow rate, (4 marks)

  ii) Velocity of the gas at the other end. (4 marks)

  Take R=287 $Jkg^{-1}K^{-1}$

b) Consider an airplane flying through still air at a velocity of 200 ms^{-1}. Assuming steady isentropic flow of a calorically perfect gas having and given that the ambient air is has a temperature of 288 K and a pressure of 101.3 kPa, determine;

  i) The Mach number and hence classify the disturbance, (4 marks)

  ii) Temperature, pressure and density at the nose of the airplane. (8 marks)

  Take $\gamma=\frac{7}{5}$ and R=287 $Jkg^{-1}K^{-1}$

QUESTION FOUR (20 MARKS)

a) Considering a one-dimensional flow of a calorically perfect ideal gas undergoing a steady flow, with no area change where viscous effects and wall friction have no time to influence the flow, heat conduction and wall heat transfer do not also have time to influence the flow, for a normal shock wave having a disturbance speed D

i) Derive the equation for the Rayleigh line. $P_2 - P_1 = \rho_1{}^2 D^2 \left[\frac{1}{\rho_1} - \frac{1}{\rho_2}\right]$ (4 marks)

ii) Derive the Rankine-Hugoniot relation $h_2 - h_1 = \frac{1}{2}(P_2 - P_1)\left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right]$ (6 marks)

b) Considering air flow through a normal shock. Given that the upstream conditions are $u_1 = 600ms^{-1}, T_{01} = 500K, P_{01} = 700kPa$, using Rankine-Hugoniot relations determine the downstream conditions M_2, u_2, T_2 and P_{02} (10 marks)

Take γ*=1.4,* $c_p = 1000Jkg^{-1}K^{-1}$ *and* $R=287\ Jkg^{-1}K^{-1}$

QUESTION FIVE (20 MARKS)

a) Consider a gas flowing over a wedge inclined at an angle of $20^0$to the horizontal. Considering a calorically perfect ideal gas. Given that the upstream conditions are $P_1 = 100kPa, T_1 = 300K$ and $M_1 = 3$. By considering Hodograph and characteristic relations, determine the downstream pressure and temperature for a shock angle of; $\beta = 37.76^0$ relating to a weak oblique shock wave, (5 marks)

b) Considering an oblique shock wave, if $\Delta\theta \rightarrow 0$, assuming a negligible entropy change relative to pressure and velocity, for an isentropic flow, derive the expression for the Prandtl-Mayer function, $\nu(M)$ (10 marks)

c) Consider a calorically perfect ideal gas. Given that it has a temperature of 300 K and is moving with a velocity of 500 ms^{-1}, determine the Prandtl-Mayer function $\nu(M)$.

$Take\ \gamma = 1.4\ and\ \ R = 287\ Jkg^{-1}K^{-1}$ (5 marks)