MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year
SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY SUPPLEMENTARY EXAMINATIONS
BACHELOR OF ECONOMICS
BACHELOR OF EDUCATION
BACHELOR OF ARTS
SMA 260: PROBABILITY AND STATISTICS I

Instructions to the Candidate:

1. *Answer **Question 1** and any other **two** questions.*
2. *You must have a Scientific Calculator and Statistical tables for this paper:*

1. (a) Outline *three* characteristics of the normal probability distribution. (3 *marks*)

   (b) The operational lifespan of a given brand of mobile phones has been found to have a normal distribution with 40.0% of the phones having a lifespan of 3.0 years or less and 12.4% of the phones having a lifespan of more than 5.5 years.

   Determine the mean and the standard deviation of the mobile phones; (4 *marks*)

   (c) A discrete random variable x has a discrete uniform probability distribution with probability mass function (*p.m.f.*) given by:

$$f(x) = \begin{cases} \frac{1}{n} & \text{for} \quad x = 0, 1, 2, 3, \ldots\ldots., n. \\ 0 \ ; & \text{elsewhere} \end{cases}$$

   Show that the variance of the random variable x is: $var(x) = \frac{1}{12}(n^2 - 1)$ (6 *marks*)

   (d) An ecologist in wildlife predators claims that three out of five successful catches by a cheater are snatched by other predators. A random sample of 12 cheater catches was taken. Assuming that this claim is true, determine the probability that among these successful catches by a cheater, the following were snatched by other predators:

   (i) between 3 and 6 catches inclusive;

   (ii) at least 4 catches. (7 *marks*)

(e) The operational lifespan of a given brand of desktop computers has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.

(i) Determine the proportion of the desktop computers that will have a lifespan of between 3.8 years and 6.6 years; (4 *marks*)

(ii) If these desktop computers have a warranty period of 2 years, determine the proportion of original sales which will require replacement through this warranty. (3 *marks*)

(iii) If the manufacturer of these desktop computers wants only 5% of the computers to be replaced through this warranty, determine the warranty period that should be set to achieve this. (3 *marks*)

2. A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) & \text{for} \quad 1 \leq x \leq 5. \\ 0 \quad ; \text{ elsewhere} \end{cases}$$

(a) Show that the function $f(x)$ is a probability density function (*p.d.f.*). (3 *marks*)

(b) Determine each of the following measures about the random variable x in the probability distribution:

(i) the mode of x

(ii) the median of x

(iii) the mean of x

(iv) the variance of x (14 *marks*)

(c) Determine the probability below from the distribution: (3 *marks*)

$P(2 \leq x \leq 4)$

3. (a) Outline *three* characteristics of the binomial probability distribution. (3 *marks*)

(b) A discrete random variable x has a probability function given by:

x	0	1	2	3	4	5
$f(x)$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Determine the mean and the variance of the random variable x in the probability distribution. (4 *marks*)

(c) Following public outcry of teenage pregnancy in secondary schools, pregnancy test was to be carried out for form four students in a certain girls' school with a total of 120 girls. A total of 12 girls were pregnant. A random sample of 30 girls was taken for testing in which the result was categorised as positive (pregnant) and negative (not pregnant). Assuming a hyper-geometric probability distribution, determine the probability that 4 of the girls sampled tested positive. (6 *marks*)

(d) It has been observed that 1 out of 40 match sticks in a match box from a production process fail to light. Assuming a Poisson probability distribution, determine the probability that among 6 such match boxes randomly selected from the production process:

(i) between 6 and 8 match sticks inclusive will fail to light;

(ii) at least 3 match sticks will fail to light. (7 *marks*)

4. The age distribution of employees working at the headquarters of a county government is found to be normally distributed with a mean of 42 years and a standard deviation of 8 years.

(a) Suppose an employee whose age is 45 years and above is allowed to retire voluntarily. Determine the proportion of employees who are allowed to retire voluntarily. (2 *marks*)

(b) Due to the burden of the huge wage bill, it is desired to reduce the present number of employees according to the following scheme:

- To retrain the employees whose age is below 36 years.
- To retain the employees whose age is between 36 and 43 years at their current workstations.
- To transfer the employees whose age is between 43 and 52 years to the national government.
- To make the employees whose age is above 52 years retire permanently.

Determine the proportion of the employees to be retrained, those to be retained, those to be and those to be transferred to the national government, and those to be retired permanently. (7 *marks*)

(c) The oldest 30% of the employees are allowed to retire voluntarily. Determine the minimum age for an employee to be allowed to retire voluntarily. (3 *marks*)

(d) Suppose the scheme in (b) is revised based on the following criteria:

- To retrain the first 15% from the lower age group.
- To retain the next 45% in their current workstations.
- To transfer the next 30% to the national government.
- To make 10% from the highest age group retire permanently.

Determine the age limits of the employees retained and those to be transferred to the national government. (8 *marks*)

5. (a) A continuous random variable x has an exponential probability distribution with probability density function (*p.d.f.*) given by:

$$f(x) = \begin{cases} \lambda\, e^{-\lambda x} & \text{for} \quad 0 \leq x \leq \infty, \quad \lambda > 0 \\ 0 \ ; & \text{elsewhere} \end{cases}$$

Show that the variance of x is given by: $var(x) = \frac{1}{\lambda^2}$ (10 *marks*)

(b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x \ ; & \text{for} \quad 0 \leq x \leq 1. \\ 0 & \text{elsewhere} \end{cases}$$

(i) Determine the probability density function of a continuous random variable $y = 8x^3$ using the change of variable technique. (5 *marks*)

(ii) Hence determine the following:

I. the mean of y ;

II. the probability $P(Y > 4)$; (5 *marks*)