

MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SMA 102 : BASIC MATHEMATICS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

Question one (COMPULSORY) (30 MARKS)

- a. Distinguish the following terms in relation to propositions
- i) Disjunction (2marks)
 - ii) Conjunction (2 marks)
 - iii) Negation (2 marks)
 - iv) Tautology (2 marks)
 - v) Contradiction (2 marks)
- b. Given that $A = \{1,2,3,4,5,6\}$ and $B = \{2,4,6,7\}$, find
- i) $A - B$ (2 marks)
 - ii) $B - A$ (2 marks)
 - iii) $A \Delta B$ (2 marks)
- c. State the following theorems
- i) De Moivre's theorem (2 marks)
 - ii) Remainder theorem (2 marks)
 - iii) Factor theorem (2 marks)
 - iv) Binomial theorem (2 marks)
- d. Determine the coefficient of the fourth and seventh terms in the expansion for $(1 + 2x)^{100}$ (4 marks)
- e. Find the modulus of the complex number $z = 4i$ (2marks)

Question two (20 marks)

- a. Prove that $\sqrt{2}$ is irrational (4 marks)
- b. In how many ways can 4 boys and 2 girls be seated in a row if the girls must not be separated? (3 marks)
- c. Express the following complex number $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ in Cartesian form. (3 marks)

- d. A polynomial has a remainder 9 when divided by $x - 3$ and a remainder -5 when divided by $2x + 1$. Find the remainder when $f(x)$ is divided by $(x - 3)(2x + 1)$ (5marks)
- e. Find the value without using a calculator
- i) 9P_3 (2marks)
 - ii) 9C_3 (2marks)
 - iii) ${}^9P_3 \div {}^9C_3$ (1mark)

Question three (20marks)

- a. Out of 60 students who exercise regularly, 25 jog, 20 play basketball and 15 swim, 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 3 people do all the three. How many students don't do any of these activities? (6 marks)
- b. Given that $z = 2 - 5i$, represent it in a well labelled Argand diagram. (4 marks)
- c. The function $f(x) = ax^3 + bx^2 + 7x + 6$ has a remainder 6 if divided by $x - 1$, and is exactly divisible by $x - 2$. Find the values of a and b and hence factorise $f(x)$ completely. (6 marks)
- d. Show using a truth table that $p \vee \sim (p \wedge q)$ is a tautology (4 marks)

Question four (20marks)

- a. In how many ways can four boys and two girls be seated in a row if
- i) the girls must not be separated. (2marks)
 - ii) the girls are separated. (2marks)
- b. In how many ways can a 7 handball players be selected from a team of 12 players to participate in a match. (2 marks)
- c. In how many ways can a class of 20 students be divided, if the class contains one set of twins who must not be separated? (2marks)

- d. Apply the remainder theorem to determine the remainder in
 $(5x^3 - 4x^2 - 3x + 6) \div (x - 2)$ (3 marks)
- e. Using long division, find the remainder in $(x^3 + 3x^2 - 13x - 10) \div (x + 5)$ (3marks)
- f. Consider the propositions
P: Mathematicians are generous
Q: spiders hate algebra
Write the compound propositions symbolized by
- i) $\sim (P \wedge Q)$
- ii) $\sim P \leftrightarrow Q$ (6 marks)

Question five (20marks)

- a. Construct the truth table for the following proposition and determine whether it is either a tautology, contradiction, or contingency.
 $[(p \rightarrow q) \vee \sim q] \rightarrow \sim p$ (4 marks)
- b. Show that $[(p \rightarrow q) \wedge \sim q] \Rightarrow \sim p$ (3 marks)
- c. Show that $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (3 marks)
- d. Find the value of c and d if $cx^4 + dx^3 - 8x^2 + 6$ has a remainder $2x + 1$ when divided by $x^2 - 1$. (5 marks)
- e. Express $\left(\frac{1+3\sqrt{2}i}{1-i}\right)^2$ in polar form (5 marks)