



MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR APRIL SESSION EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 300: REAL ANALYSIS

DATE: 22/7/2021

TIME: 8:30 – 10:30AM

INSTRUCTIONS

Answer question one and ANY TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Define the following terms
- i) Set
 - ii) Subsets complement
 - iii) Universal set
 - iv) Function
 - v) Rational numbers (5 marks)
- b) Show that $\sqrt{2}$ is irrational (4 marks)
- c) The square of an odd integer is odd and the square of an even integer is even proof (4 marks)
- d) Prove that the function $f(x) = x^2$ is continuous at every point $a \in R$ (4 marks)
- e) Show that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ is convergent (3 marks)
- f) Given that a and b are rational with $b \neq 0$ and s is an irrational number such that $a - bs = t$. Show that t is irrational hence show that $\frac{-1+\sqrt{3}}{1+\sqrt{3}}$ is irrational (4 marks)
- g) i) Show that the infinite set $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$ is bounded (3 marks)
- ii) Determine the supremum and the infimum of the set in question g i) above (3 marks)

QUESTION TWO 20 MARKS

- a) Let π be an arbitrary indexing set and G_α be a collection of open subsets of a metric space X . Prove that $\bigcup G_\alpha$ is also open in X . (5 marks)
- b) Let (X, ∂) be a metric space and $A \subset X$. Then if P is a limit point of A . Prove that every nbd of P contains infinitely many points distinct from P . (5 marks)
- c) Let (X, ∂) be a metric space. Prove that for any subset A of X with a finite number of elements cannot have limit point. (5 marks)
- d) Prove that the intersection of an arbitrary family of closed sets is closed and the union of a finite number of closed sets is closed. (5 marks)

QUESTION THREE 20 MARKS

- a) Show from the first principle that
 - i) $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$
 - ii) $\lim_{n \rightarrow \infty} \frac{3n+2}{n+1} = 3$ (5 marks each)
- b) Show that the sequence $\frac{1}{2^n}$ in $\mathbb{R}$ is a Cauchy sequence. (5 marks)
- c) Proof that $\sqrt{3}$ is irrational (5 marks)

QUESTION FOUR 20 MARKS

- a) If $\{A^n\}$ is a collection of countable sets, then $A = \bigcup A^n$ is countable (5 marks)
- b) Prove that a subset of a countable set is countable. (5 marks)
- c) Prove that the empty set is open. (5 marks)
- d) Prove that set of all even natural numbers is countable. (5 marks)

QUESTION FIVE 20 MARKS

- a) Prove that the $f(x) = x^2 + 2x + 6$ is continuous at $x=3$ (5 marks)
- b) Show that the function $f(x) = \begin{cases} \frac{3x^2-2x-8}{x-2} & , x \neq 2 \\ 8 & x=2 \end{cases}$

Is discontinuous at $x=2$. redefine the function to make it continuous at $x=2$ (5 marks)

- c) Show that if $r = \sqrt{n+1} - \sqrt{n-1}$ for any integer $n > 1$ then r is rational (5 marks)
- d) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, determine whether the series converges or diverges using the integral test. (5 marks)