# MACHAKOS UNIVERSITY

**University Examinations 2018/2019**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE**

**FOURTH YEAR SECOND SEMESTER EXAMINATION FOR**

**BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)**

**BACHELOR OF SCIENCE (MATHEMATICS)**

**BACHELOR OF EDUCATION**

**BACHELOR OF ARTS**

**SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II**

**DATE: 30/4/2019** **TIME: 8:30 – 10:30 AM**

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**INSTRUCTIONS:**

Attempt question ***one (compulsory***) and any other ***two questions***.

**QUESTION ONE (Compulsory) (30MARKS)**

a) Define a second order linear partial differential equation (3 marks)

b) Define a Fourier transform of a function $f(x)$ (4 marks)

c) Consider the convolution theorem

$$F^{-1}\left[\overline{F}(\xi)\overline{G}(\xi)\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)g(x-u)du$$

Prove the theorem. (4 marks)

d) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.

i) Classify the equation (4 marks)

ii) Determine its characteristic equation (5 marks)

iii) Transform the equation to canonical form (5 marks)

iv) Solve the equation (5 marks)

**QUESTION TWO (20 MARKS)**

An infinitely long string having one end at $x = 0$ is initially at rest on the $x - axis$. At $t = 0$ the end $x = 0$ begins to move along the $u - axis$ in a manner described by $u(0,t) = a\cos\sigma t$. Determine the displacement $u(x,t)$ of the string at any point at any time.

**QUESTION THREE (20MARKS)**

Solve the wave equation $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$

Given that $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.

**QUESTION FOUR (20MARKS)**

Determine the characteristic of the Tricomi equation $yu_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$. Put this equation into canonical form in the upper half- plane $y > 0$.

**QUESTION FIVE (20 MARKS)**

Determine the Fourier transforms for the following functions;

i) $f(x) = e^{-a|x|}$ (10 marks)

ii) $f(x) = e^{-ax^2}$ (10 marks)