MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (ACTUAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 200 CALCULUS II

DATE: 8/12/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the area of the rectangle bounded by the line $y = 2$, the $x - axis$, and ordinates $x = 1 \quad and \quad x = 5$ (4 marks)

b) The path traced by a moving ant in half of an hour is described by an arc from $\theta = 0 \quad to \quad \theta = \frac{\pi}{4}$ of the curve given by $x = 3\cos\theta \; and \; y = 3\sin\theta$. Calculate the velocity of the ant in the time of its motion. (4 marks)

c) A fundamental theorem of integral calculus advocates that the sum or difference of two or more functions is equal to the sum or difference of their integrals. Taking two functions $f(x) \quad and \quad F(x)$ and considering the conventional meanings of integral calculus prove this theorem. (4 marks)

d) Evaluate the improper integral $\int_{-\infty}^{-1} \frac{1}{x^4} dx$ (4 marks)

e) Determine

i. $\int \cos^3 x \sin x dx$ (4 marks)

ii. $\int x^2 e^{3x} dx$ (5 marks)

iii. $\int \frac{x-11}{(x+3)(x-4)} dx$ (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the integral $\int x^n e^{-x} dx$

i. Prove that $\int x^n e^{-x} dx = -x^n e^{-x} + n\int x^{n-1} e^{-x} dx$ (3 marks)

ii. Use the formula obtained in (i) above to evaluate $\int_0^\infty x^n e^{-x} dx = n!$; where n is a positive integer (5 marks)

b) Consider the curve $y = \frac{2}{x^2+1}$ between $x=0$ *and* $x=1$; Subdividing the interval into 10 equal parts;

i. Sketch the curve (3 marks)

ii. Use the Simpson's rule with 10 intervals to approximate the value of $\int_0^1 \frac{2}{x^2+1} dx$ (3 marks)

iii. Calculate the exact value of $\int_0^1 \frac{2}{x^2+1} dx$ by integration method (3 marks)

iv. Determine the percentage error generated by working out $\int_0^1 \frac{2}{x^2+1} dx$ using the Simpson's rule with the ten intervals (3 marks)

QUESTION THREE (20 MARKS)

a) Show that the curved surface area of a cone of radius $r\ units,\ \ height\ h,\ \ and\ \ slant\ \ height\ \ l$ is given by $S.A = \pi\, rl$ (6 marks)

b) Use the substitution method to prove $\int \sqrt{x^2 - 2^2}\ dx = \frac{x}{2}\sqrt{x^2-4} - 2\cos^{-1}\left(\frac{x}{2}\right) + c$ .Where c is the constant of integration. (7 marks)

c) Consider the region bounded by the equations $y = \sin x\ \ and\ \ y = 0$ from the line $x = 0\ \ to\ \ x = \pi$. Calculate the centroid center of mass) of the region. (7 marks)

QUESTION FOUR (20 MARKS)

Evaluate;

a) $\int \frac{e^x}{1+e^x}\ dx$ (2 marks)

b) $\int \frac{dx}{\sqrt{9+x^2}}$ (4 marks)

c) $\int \sin\left(\log x\right)\ dx$ (7 marks)

d) $\int \frac{dx}{(x-1)^2(x-2)(x^2+4)}$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Prove the following integrals;

i. $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}}\, dx\ = -\left(\sqrt{1-x^2}\right)\sin^{-1} x + x + c$ (5 marks)

ii. $\int \cos ec^n x dx = -\frac{\cot x\ \cos ec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \cos ec^{n-2} x dx + c$ (6 marks)

Where c is the constant of integration.

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$; where n is a positive integer (9 marks)