MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR DEGREE IN

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (BUILDING AND CIVIL ENGINEERING)

ECU 203: ENGINEERING MATHEMATICS VIII

Date: 12/6/2017 **Time: 8:30 – 10:30 AM**

INSTRUCTIONS

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Solve the equation $\frac{\partial^2 u}{\partial x \partial y} = \sin(x+y)$ given that $\frac{\partial u(x,0)}{\partial x} = 1$ and $u(0,y) = (y-1)^2$

(5 marks)

b) Evaluate $\int (t^2+4) \cdot \delta(t-2)dt$ (5 marks)

c) Determine the polynomial solution to the Legendre equation

$$(1-x^2)y'' - 2xy' + 12y = 0$$ (5 marks)

d) Evaluate $\int J_5(x)dx$ (5 marks)

e) Determine the Laplace transform of $f(t) = e^{at}$ where a is a constant. (5 marks)

f) Determine the half range sine Fourier series representing the function $f(x)$ defined by;

$f(x) = 1 + x \quad ; \quad 0 < x < \pi$

$f(x) = f(x + 2\pi)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Determine the inverse of $\dfrac{5s+1}{(s-4)(s+3)}$ (3 marks)

b) Use Laplace transforms method to solve the second order differential equation

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=2e^{3t} \text{ given that at } t=0\ ;\ x=5 \quad and \quad \frac{dx}{dt}=7$$ (7 marks)

c) Calculate $P_4(x)$ using the Rodrigues formula for the Legendre functions. (7 marks)

QUESTION THREE (20 MARKS)

a) Use the Bessel's function identities to express $J_3(x)$ in terms of

$$J_0(x) \ and \ J_1(x)$$

(3 marks)

b) Evaluate $\int x^3 J_0 dx$ (3 marks)

c) Determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x \ and \ \cos x$ (4 marks)

d) Show that $xF(1,1,2;-x)=\ln(1+x)$ (4 marks)

e) Determine the general solution to Bessel's equation of order zero. (6 marks)

QUESTION FOUR (20 MARKS)

a) Solve $\dfrac{\partial^2 u}{\partial x \partial y}=\sin x \sin y$ subject to the conditions;

$$\frac{\partial u}{\partial x}\left(x,\frac{\pi}{2}\right)=2x$$

$$u(\pi, y)=2\sin y$$ (5 marks)

b) Use the method of separation of variables to find the solution of the equation;

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{k}\frac{\partial u}{\partial t}$$ which satisfies the conditions;

i.) $u(0,t)=u(a,t)=0 \quad \forall\ t\geq 0$

ii.) $u(x,0)=x \quad for\ 0\leq x\leq a$ (15 marks)

QUESTION FIVE (20 MARKS)

a) i) Define a Dirac delta function (1 mark)

ii) State the convolution theorem of Fourier transforms. (3 marks)

iii) Given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(x) g(t-x) dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t) = u(t)$ and $g(t) = \begin{cases} \sec^2 t & |t| < \dfrac{\pi}{4} \\ 0 & otherwise \end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t) = \dfrac{1 + \tan^2 t}{1 + \tan t}$ (4 marks)

b) Determine the Fourier's series expansion for;

$$f(x) = \begin{cases} 0 & ; -\pi \leq x < \dfrac{-\pi}{2} \\ \cos x & ; \dfrac{-\pi}{2} \leq x < \dfrac{\pi}{2} \\ 0 & ; \dfrac{\pi}{2} \leq x < \pi \end{cases}$$ (4 marks)

c) Determine a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$ (4 marks)

d) Evaluate the inverse transform of $f(\omega) = \dfrac{1}{2\pi(a + j\omega)^2}$ where $a > 0$ (5 marks)