MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: 19/01/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Solve the initial value problem $\frac{d^2y}{dx^2}+y=0$; $y(0)=3$, $y'(0)=-4$ (5 marks)

b) Consider the initial value problem

$$\frac{d^3z}{dt^3}-3\frac{d^2z}{dt^2}+2\frac{dz}{dt}-z=\sin t \qquad z(0)=1\ ,\ z'(0)=0,\ z''(0)=3$$

Reduce the equation to a system of first order linear differential equations. (5 marks)

c) Consider the initial – value problem $\frac{dy}{dx}=x^2+y^2$, $y(1)=3$. Determine whether the differential equation has a unique solution (5 marks)

d) Solve the system of differential equations below using the matrix method

$$\frac{dx_1}{dt}=3x_1-2x_2$$
$$\frac{dx_2}{dt}=-x_1+2x_2$$

(5 marks)

e) Determine the integrability of the differential equation $zdx+zdy+2(x+y+\sin z)dz=0$

(5 marks)

f) Consider the differential equation $x^2(1-x)y'' + (1-x)y' + y = 0$

i. Determine the singular points (2 marks)

ii. Determine the nature of the singular points in (i) above (3 marks)

QUESTION TWO (20 MARKS)

Use the method of Frobenius to solve the differential equation

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

QUESTION THREE (20 MARKS)

Consider the system of differential equation

$$\frac{dx}{dt} = -2x + 2x^2$$
$$\frac{dy}{dt} = -3x + y + 3x^2$$

a) Determine the critical points

b) State the nature of the critical point

c) Linearize the system

d) Solve the system for x *and* y

QUESTION FOUR (20 MARKS)

Use the Netani's method to solve the following differential equations

a) $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$ (9 marks)

b) $2zdz + dy + (2x^2 + 2xy + 2xz^2 + 1)dx = 0$ (11 marks)

QUESTION FIVE (20 MARKS)

Solve the Legendre differential equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$