



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 101 FUNDAMENTALS OF ACTUARIAL MATHEMATICS

DATE: 26/8/2022

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) You are given a discount function v where $v(1, 3) = 0.9$, $v(3, 6) = 0.8$, $v(8, 6) = 1.2$
- How much must you invest at time 1, in order to accumulate 10 at time 8? (3 marks)
 - If you invest 100 at time 3, how much will have accumulated by time 8? (3 marks)
- b) You are given that $q_{60} = 0.20$, $q_{61} = 0.25$, $q_{62} = 0.25$, $q_{63} = 0.30$, $q_{64} = 0.40$.
- Find ℓ_x for ages 60 – 65, beginning with $\ell_{60} = 1000$. (3 marks)
 - Find the probabilities of the following:
 - (61) will die between the ages of 62 and 64. (3 marks)
 - (62) will live to age 65. (3 marks)
- c) Given that $e_{65} = 0.8$, find e_x for $x = 60 - 64$ (4 marks)
- d) A group of individuals age 40, each invest 1000 in a fund earning interest at 5%. At the end of 20 years the fund is divided equally among the survivors. If $20P_{40} = 0.8$, how much does each get? (4 marks)
- e) Find the price of a Kshs. 1,000 par value 10 – year bond with coupons at 8.4% payable semi-annually, which will be redeemed at Kshs. 1050. The bond is bought to yield 10% convertible semi-annually. Use the basic formula. (4 marks)
- f) Calculate the time in days for £1,500 to accumulate to £1,550 at
- Simple rate of interest of 5% per annum (3 marks)
 - A force of interest of 5% per annum. (3 marks)

QUESTION TWO (20 MARKS)

- a) A loan of £50,000 is to be repaid over 5 years by a level annuity payable annually in arrears. The amount of the annual payment is calculated on the basis of an interest rate of 3% per annum effective. Immediately after the third repayment was made, the borrower requests that he be able to pay off the loan with a single lump sum. Use a repayment schedule to calculate the value of the lump sum required to repay the loan at this time. (6 marks)
- b) Differentiate between the following ways of calculating the outstanding loan:
- i. Prospective approach. (2 marks)
 - ii. Retrospective approach (2 marks)
- c) Person *A* borrows \$10,000 from Person *B* and agrees to repay it with equal quarterly instalments of principal and interest at 8% convertible quarterly over six years. At the end of two years Person *B* sells the right to receive future payments to Person *C* at 10% convertible quarterly. Find the total amount of interest received:
- i. By Person *C*. (5 marks)
 - ii. By Person *B*. (5 marks)

QUESTION THREE (20 MARKS)

- a) An investor buys an n – year annuity with a present value of Kshs. 1,000 computed at 8%. The investor pays a price which will permit the replacement of the original investment in a sinking fund earning 7% and will also produce an overall yield rate of 9% on the entire transaction. Find the price which the investor should pay for the annuity. (10 marks)
- b) A loan of £10,000 is to be repaid over 10 years by a level annuity payable monthly in arrears. The amount of the monthly payment is calculated on the basis of an interest rate of 1% per month effective. Find:
- i. The monthly repayment, (4 marks)
 - ii. The total capital repaid and interest paid in:
 - I. The first year and (3 marks)
 - II. The final year, (3 marks)

QUESTION FOUR (20 MARKS)

- a) An asset is being depreciated over a 10-year period. It has no salvage value at the end of the ten years, that is $S=0$. If the depreciation charge in the third year is Kshs. 1000, find the depreciation charge in the ninth year:
- i. By the sinking fund method, assuming $j=0.05$. (3 marks)
 - ii. By the straight-line method. (2 marks)
 - iii. By the sum of the years digit method. (3 marks)
- b) Why can the declining balance method not be used? (1 mark)
- c) Find the original value of the asset in each of the above cases in (a) (7 marks)
- d) Define the terms:
- i. Life annuity (1 mark)
 - ii. Life insurance (1 mark)
 - iii. Life tables (1 mark)
 - iv. The theory of Consistency (1 mark)
 - v. The probability of surviving (1 mark)

QUESTION FIVE (20 MARKS)

- a) A loan is repaid with repayments which start at \$200 the first year and increase by \$50 per year until a payment of \$1,000 is made, at which time payments cease. If interest is 4% effective, find the amount of principal in the fourth payment. (6 marks)
- b) An investor in common stock measures investment returns annually using an effective rate of interest. The investor earns 15% during the first year, -5% during the second year and 8% during the third. Find the equivalent level effective rate of return over the three-year period. (4 marks)
- c) Find the present value of \$5,000 to be paid at the end of 25 months at a rate of discount of 8% convertible quarterly:
- i. Assuming compound discount throughout. (3 marks)
 - ii. Assuming simple discount during the final fractional period. (4 marks)
- d) Find the level effective rate of interest over a three-year period which is equivalent to an effective rate of discount of 8% the first year, 7% the second year, and 6% the third year. (3 marks)