MACHAKOS UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF EDUCATION

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SST 201, SMA 362: OPERATIONS RESEARCH I

Instructions to the Candidate:

1. *Answer* ***Question* 1** *and any other* ***two*** *questions.*
2. *You must have a Scientific Calculator and Graph paper for this paper:*

1. (a) Outline *four* conditions which must be satisfied for a problem to be solved using linear programming technique. (4 *marks*)

(b) Explain each of the following types of variables as used in linear programming:

(i) Slack variable;

(ii) Surplus variable. (4 *marks*)

(c) A factory produces four types of electrical cables – A, B, C and D. The factory wishes to maximise its weekly contribution to revenue. The sale prices per roll of cable are 20, 25, 30 and 60 in thousand Kenya shillings respectively. The factory employs 200 skilled machines, 150 unskilled, and 100 casual employees who work for 40 hours in a week. The time in hours it takes to produce one roll of each type of cable is as shown in the table below:

Component		A	B	C	D
Hours	Skilled	2	6	3	5
	Unskilled	5	8	4	2
	Casual	4	7	8	6

Formulate a linear programming model for this problem. (4 *marks*)

(e) A dairy production plant produces two products - cheese and milk. Cheese costs Ksh 60 per unit while milk costs Ksh 45 per unit. The manager of the firm wants to determine the daily production plan which minimises the cost of production. The production constraints are as spelt out in the table below:

Labour cost	Machine cost	Packaging material

Cheese	2	2	1
Milk	3	1	1
Minimum cost	180	120	80

The Ministry of Health has issued a directive that a minimum of 20 units of cheese must be produced per day in conformity with a health strategy.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the graphical method, determine the optimum production plan which minimises the cost of production for the dairy plant. (6 *marks*)

(iii) Interpret the optimum solution obtained in (ii) above. (3 *marks*)

(e) A shipping company is required to meet the demands at various destinations by transporting 80, 100, 40, 120, 60 containers respectively from supplies 120, 80, 160, 40 containers from various sources. The transport cost in thousand Kenya shillings per unit of container over the various routes are as shown in the matrix below:

$$C = \begin{bmatrix} 4 & 8 & 2 & 5 & 10 \\ 3 & 6 & 1 & 4 & 2 \\ 6 & 2 & 8 & 3 & 5 \\ 5 & 1 & 2 & 4 & 3 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the containers at the cheapest cost possible. Determine the initial basic feasible solution using each of the following methods:

(i) Northwest corner (NWC) rule.

(ii) Vogel's approximation method (VAM). (6 *marks*)

2. A lamp production plant produces two products: florescent tubes and energy saving bulbs. Tubes contribute Ksh 60 per unit while bulbs contribute Ksh 80 per unit. The manager of the firm wants to determine the daily production plan which maximises contribution to revenue. The production constraints are as shown in the table below:

	Labour hours	Machine hours	Glass material	Coating material
Tubes	3	5	25	1
Bulbs	5	4	18	1
Maximum available	900	1200	4500	200

The Ministry of Energy has issued a directive that a minimum of 40 bulbs must be produced per day in conformity with an energy saving strategy.

(i) Formulate a linear programming model for the problem. (4 *marks*)

(ii) Using the graphical method, determine the optimum production plan which maximises revenue for the firm. (12 *marks*)

(iii) Interpret the optimum solution obtained in (ii) above. (4 *marks*)

3. A restaurant sells three types of hot drinks – tea, cocoa and coffee. The sale prices per cup of the drinks are: tea Ksh 40, cocoa Ksh 25 and coffee Ksh 50. The restaurant wishes to determine the daily production plan which maximises contribution to revenue. The daily production plan has been modelled as an LP program as shown below.

Maximise :	$z = 40x_1 + 25x_2 + 50x_3$	
Subject to :	$2x_1 + 3x_2 + x_3 \leq 1200$	Fuel
	$x_1 + x_3 \leq 450$	Labour hours
	$2x_1 + 4x_3 \leq 600$	Sugar
	$x_2 \leq 150$	Agreement
With :	$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$	Non-negativity conditions

where x_1 represent the number of cups of tea produced,
x_2 represent the number of cups of cocoa produced, and
x_3 represent the number of cups of coffee produced.

(i) Using the Simplex method, solve the linear programming model, and hence determine the daily production plan which maximises contribution to revenue for the restaurant. (16 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (4 *marks*)

4. Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise : $z = 50x_1 + 80x_2 + 90x_3$

Subject to : $x_1 + x_2 + 2x_3 \geq 30$

$x_1 + 2x_2 + x_3 \geq 40$

With : $x_1,\ x_2,\ x_3 \geq 0$

(i) Determine the symmetrical dual of the linear programming model above. (4 *marks*)

(ii) Using the Simplex method, determine the optimum solution of the dual program for the linear programming problem above. (10 *marks*)

(iii) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (6 *marks*)

5. A cement distribution company is required to meet the demands 50, 70, 100, 60 in thousand bags at various destinations from supplies 100, 60, 120 in thousand bags from various sources. The transport cost per unit in thousand Kenya shillings over the various routes are as given in the cost matrix below:

$$C = \begin{bmatrix} 5 & 1 & 3 & 4 \\ 2 & 4 & 2 & 5 \\ 1 & 3 & 2 & 4 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the cement at the cheapest cost possible. Using the northwest corner (NWC) rule:

(i) Determine the optimal solution that minimises the cost of transport; (17 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (3 *marks*)