



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 106: ENGINEERING MATHEMATICS III

DATE: 17/4/2023

TIME: 11:00 – 1:00 PM

INSTRUCTIONS

Answer question **ONE (Compulsory)** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

- a) Given $\vec{a} = (4, 0, 3)$ and $\vec{b} = (-2, 1, 5)$, find $|2\vec{a} + 5\vec{b}|$. (3 marks)
- b) Find the unit vector in the direction of $\vec{p} = 8\hat{i} - \hat{j} + 4\hat{k}$ (3 marks)
- c) Convert the rectangular coordinates $(-1, 1, \sqrt{6})$ to;
- i. Cylindrical coordinates (3 marks)
- ii. Spherical coordinates (3 marks)
- d) Find the equation of the plane containing the points $P = (-1, 4, 2)$, $Q(6, -2, 8)$ and $R(5, -1, -1)$. (5 marks)
- e) Solve the system below using Gaussian – elimination method, (4 marks)
- $$\begin{aligned} 2x + y &= 3 \\ x &= 2y \end{aligned}$$
- f) Determine characteristic equation of the matrix $R = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$ (5 marks)
- g) Using truth table show that $X + \bar{X}Y = X + Y$. (4 marks)

QUESTION TWO (20 MARKS)

- a) Show that $\vec{A} \times \vec{B}$ is orthogonal to both $\vec{A}$ and $\vec{B}$ (5marks)
- b) Determine the vector equation and parametric equation of the line through $(1, -5, 6)$ and parallel to $-\hat{i} + 2\hat{j} - 3\hat{k}$. (5marks)
- c) Find the equation of the plane through $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to $\hat{n} = \hat{i} - \hat{j} + 2\hat{k}$ and sketch the plane. (5 marks)
- d) Given $\vec{v} = 4\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{u} = \hat{i} - \hat{j} + 2\hat{k}$ then find $Comp_{\vec{u}} \vec{v}$ and $Proj_{\vec{u}} \vec{v}$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Solve the system of linear equations below using Gauss – Jordan elimination method
 $-3x + 6y + 8z = -5$ (7 marks)
 $2x + 4y - 3z = 8$
 $x - 3y - 2z = 6$
- b) Evaluate the determinant of $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix}$ using third row cofactor expansion. (4 marks)
- c) Determine the inverse of $Q = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ using row reduction method (9 marks)

QUESTION FOUR (20 MARKS)

- a) Compute the eigenvalues and the corresponding eigenvectors of $T = \begin{pmatrix} 3 & 2 \\ 3 & -2 \end{pmatrix}$ (10 marks)
- b) Diagonalize matrix $T = \begin{pmatrix} -1 & 4 \\ 0 & 3 \end{pmatrix}$ (10 marks)

QUESTION FIVE (20 MARKS)

- a) Using truth table proof the dual distributive property of Boolean Algebra
 $X + YZ = (X + Y) \cdot (X + Z)$ (5 marks)
- b) Given $H = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ find the diagonal matrix D , D^2 and H^2 . (15 marks)