



# MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 230: VECTOR ANALYSIS

DATE:

TIME:

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## INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

### QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Consider the region bounded by the equations  $y = \sin x$  and  $y = 0$  from the line  $x = 0$  to  $x = \pi$ . Calculate the centroid (center of mass) of the region (3 marks)
- b) Calculate the area bounded by the curve  $y = \frac{1}{\sqrt{x}}$ ,  $x$ -axis and the line  $x = 1$  (3 marks)
- c) Evaluate  $\int \left( \frac{2x^3 + 5x^2 + 8x + 9}{x} \right) dx$  (4 marks)
- d) Find the length of the arc from  $\theta = 0$  to  $\theta = \frac{\pi}{4}$  of the curve given by  $x = 3 \cos \theta$ ,  $y = 3 \sin \theta$ .
- e) Evaluate the definite integral  $\int_0^2 \cos^2 x dx$  (5 marks)
- f) Sketch the curve  $y = 2x^2$  from  $x = 0$  to  $x = 5$ . Hence evaluate the area enclosed by the curve,  $x$ -axis and the ordinates  $x = 1$  to  $x = 4$  (5 marks)
- g) Find the area between the curves  $x^2 = y$  and the straight line  $y = 2$  (5 marks)

- h) Show that the improper integral  $\int_1^{\infty} \frac{1}{x^2} dx$  is convergent (5 marks)

**QUESTION TWO (20 MARKS)**

- a) Evaluate the improper integral  $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$  (3 marks)
- b) Evaluate by adjustment of the derivative  $\int \frac{x^2 + x + 1}{2x^3 + 3x^2 + 6x + 5} dx$  (4 marks)
- c) Use appropriate technique to evaluate the integral  $\int \sin^4 x dx$  (6 marks)
- d) Use partial fractions to evaluate the integral  $\int \frac{3x^2 - 2x - 7}{x^2 - x - 2} dx$  (7 marks)

**QUESTION THREE (20 MARKS)**

- a) Prove that  $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$  where  $a$  and  $b$  are constants (4 marks)
- b) Use the fact that  $\sin^2 x + \cos^2 x = 1$  to evaluate the integral  $\int \cos^3 x \sin^2 x dx$  (4 marks)
- c) A particle carrying a positive electrical charge  $+q$  is released at a distance  $d$  from an immovable positive charge  $+Q$ . Calculate amount of work done on the particle as its distance increases to  $2d$ . (4 marks)
- d) Find the surface area generated by the loop of the curve  $x = t^2, y = t - \frac{t^3}{3}$  about the axis. (8 marks)

**QUESTION FOUR (20 MARKS)**

- a) Use integration by parts to evaluate
- i.  $\int x^3 \ln x dx$  (5 marks)
- ii.  $\int x^2 e^{2x} dx$  (5 marks)

b) Draw the curve  $6y^2 = x(x-2)^2$  and calculate the length of the loop of the curve

$$y^2 = \frac{1}{6}x(x-2)^2 \quad (5 \text{ marks})$$

c) Find the volume obtained by rotating the area under the curve  $y = 1 + x$  between  $x = 1$  and  $x = 2$  about the axis of  $x$ . (5 marks)

### QUESTION FIVE (20 MARKS)

a) Evaluate;

i.)  $\int_0^{\pi} x \sin^6 x \cos x^4 x dx$  (3 marks)

ii.)  $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta$  (6 marks)

b) Compute  $\int_0^1 \frac{\tan^{-1} x}{x^{\frac{3}{2}}} dx$  using Simpson's rule with 10 *equal* intervals. (11 marks)