MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 231: DYNAMICS I

DATE: 24/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Define the following terms as used in theoretical Dynamics;

i.) Torque (3 marks)

ii.) Dumped forces (2 marks)

iii.) Potential energy (2 marks)

iv.) Impulsive forces (3 marks)

b) A constant force $\vec{F}$ acting on a particle of mass m units changes the velocity from v_1 *to* v_2 in time c. Prove that $\vec{F} = \frac{m(\vec{v}_2 - \vec{v}_1)}{c}$ (5 marks)

c) A particle P of unit mass moves in a force field given by $\vec{F} = (3t^2 - 4t)\hat{i} + (12t - 6)\hat{j} - (6t - 12t^2)\hat{k}$, where t is time. If the velocity at $t = 1\sec$ *is* $4\hat{i} - 5\hat{j} + 10\hat{k}$, calculate the velocity at $t = 2\sec$ (5 marks)

d) A particle of mass M units move along a space curve defined by $\vec{r} = a\cos\omega t\, i + b\sin\omega t\, j$. Determine its angular momentum about the origin.(5 marks)

e) A particle is projected from apoint O on a plane inclined at an angle α with the horizontal. The particle hits the plane at right angles at A, where OA is a line of greatest slope, O being lower than A. if the particle is projected at an angle θ to the horizontal, determine the relationship between α and θ. (5 marks)

QUESTION TWO (20 MARKS)

A body of mass m units travelling in a straight line is supplied with constant power P and is subjected to resistance Mkv^2 where v is its speed and k is a constant. Show that;

i) the speed of the body cannot exceed a certain value (8 marks)

ii) if it starts from rest it acquires half the maximum speed after travelling a distance $\frac{1}{3}k\ln\left(\frac{8}{7}\right)$ (6 marks)

iii) if the power is then cut off and an additional retarding force of constant value $\vec{F}$ is imposed, calculate the subsequent time which elapses before the body comes to rest. (6 marks)

QUESTION THREE (20 MARKS)

a) A particle of mass 2 *units* moves along the $x-axis$ attracted towards the origin O by a force whose magnitude is numerically equal to $8x$. It is initially at rest at $x=20$, work out;

i.) The differential equation and initial condition describing the motion.(5 marks)

ii.) Position of particle at any time (4 marks)

iii.) The speed and velocity at any time (3 marks)

iv.) The period (2 marks)

v.) The frequency (2 marks)

b) Suppose that a particle has constant mass and that at times t_1 *and* t_2 it is located at p_1 *and* p_2 respectively and moving with velocities $\vec{v}_1=\frac{d\vec{r}_1}{dt}$ *and* $\vec{v}_2=\frac{d\vec{r}_2}{dt}$ respectively; prove that the total work done in moving the particle a long a curve from p_1 *to* p_2 is given by;

$$W=\int_c \vec{f}\cdot d\vec{r}=\frac{1}{2}m\left(\vec{v}_2^{\,2}-\vec{v}_1^{\,2}\right)$$ (4 marks)

QUESTION MARKS (20 MARKS)

a) A particle P of mass M units slides without rolling down a frictionless inclined plane AB of angle α with the horizontal. If it starts from rest at the top A of inclined plane, calculate the particle's;

i.) Acceleration (5 marks)

ii.) Velocity (4 marks)

iii.) Distance (5 marks)

b) A bullet is fired at an angle of 45^0 with the horizontal, with a velocity of $273 m/s$.

Determine

i.) the maximum height the bullet rises (3 marks)

ii.) the horizontal range R (3 marks)

QUESTION FIVE (20 MARKS)

An object of mass $40kg$ moves in the $x-y$ *plane* so that its position vector is

$\vec{r} = a\cos\omega t\hat{i} + b\sin\omega t\hat{j}$ where a, b *and* ω are positive constants and $a > b$. Determine;

i) The kinetic energy at $x = a$ *and* $y = b$ (10 marks)

ii) The work done by the force field in moving the particle from $x = a$ *to* $y = b$ (10 marks)