MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER SPECIAL/ SUPPLEMENTARY EXAMINATION FOR BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS) BACHELOR OF SCIENCE (MECHANICAL ENGINEERING) BACHELOR OF CIVIL ENGINEERING

ECU 300: ENGINEERING MATHEMATICS IX

DATE: 29/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer Question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Given that $\vec{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$

i) Show that $\vec{F}$ is a conservative force field. (2 marks)

ii) Find the scalar potential (3 marks)

b) Evaluate $\oint_C \vec{F}.d\vec{r}$ around a closed curve C bounded by $y^2 = x$ and $x^2 = y$, if $\vec{F} = (x - y)i + (x + y)j.$ (5 marks)

c) Find the area of the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$ (5 marks)

e) Verify the divergence theorem for $\vec{F} = 4xi - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4,$ $z = 0$ and $z = 3$ (8 marks)

f) Evaluate the line integral $\oint xydx + (2x - y)dy$ round the region bounded by the curve $x^2 = y$ and $x = y^2$ by use of Green's theorem. (7 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the double integral $\iint_R \frac{x}{y}\,dxdy$ where R is the rectangle $|x| \leq 1,\ 1 \leq y \leq 2$

(8 marks)

b) Verify Stoke's Theorem for the vector function $F = yi + zj + xk$, for the surface S where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary (12 marks)

QUESTION THREE (20 MARKS)

a) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2i - 3j + 4k$, $\vec{B} = i + 2j - k$ and $\vec{C} = 3i - j + 2k$ (5 marks)

b) Prove that the diagonals of a parallelogram bisect each other (5 marks)

c) A vector field $\bar{F} = yi + 2j + k$ exists over the surface S defined by $x^2 + y^2 + z^2 = 9$ bounded by $x = 0, y = 0, z = 0$ in the first octant.
Evaluate $\int \bar{F} o d\bar{S}$ over the indicated surface. (10 marks)

QUESTION FOUR(20 MARKS)

a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2i - j - 2k$ (4 marks)

b) Find the total work done in moving a particle in a force field given by $\bar{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^2$ from $t = 1$ to $t = 2$. (5 marks)

c) Prove that $\bar{C}^2 = \bar{A}^2 + \bar{B}^2 - 2\bar{A}\bar{B}\cos\theta$ where $\bar{A}, \bar{B}, \bar{C}$ are the three sides of a triangle with θ the angle between the vectors $\bar{A}$ and $\bar{B}$ (5 marks)

d) Represent $\vec{F} = zi - 2xj + yk$ in cylindrical coordinates. Thus determine F_ρ, F_ϕ, F_z.

(6 marks)

QUESTION FIVE (20 MARKS)

a) A vector field $\vec{F} = yi + 2j + k$ exists over a surface S defined by $x^2 + y^2 + z^2 = 9$ bounded by $x = 0,\ y = 0,\ z = 0$ in the first octant. (10 marks)

b) If $\vec{A} = 2i - 3j + 4k$, $\vec{B} = i - 2j - 3k$ and $\vec{C} = 2i + j + 2k$ then show that the scalar triple product in absolute represents the volume of a parallelepiped with sides $\vec{A}, \vec{B}$ *and* $\vec{C}$

(10 marks)