



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF SCIENCE (MATHEMATICS)

SPH 103: MATHEMATICS FOR PHYSICS

DATE: 19/4/2023

TIME: 2:00 – 4:00 PM

INSTRUCTIONS:

Answer question **ONE** which is compulsory and any other **TWO**

Take acceleration due to gravity, $g = 10 \text{ N/kg}$

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS)

- (a) (i) Define an odd function. (2 marks)
- (ii) A function is defined as $f(x) = \frac{x-5}{x+2}$
Find the domain, range and inverse of the function $f(x)$. (5 marks)
- (b) Given that $f(x) = 2x - 1$ and $(x) = \frac{1}{x-1}$. Find the two corresponding composite functions, giving their domains. (6 marks)
- (c) Define an irrational number hence show that $\sqrt{3}$ is irrational (5 marks)
- (d) Show that area, $A(x)$ of the region between $f(x)$ and the x – axis on the interval $[a, x]$ is given by the anti-derivative of $f(x)$ (6 marks)
- (e) From first principles, differentiate $(x) = x^3$, hence evaluate $f'(1)$ and $f'(3)$ (6 marks)

SECTION B (ATTEMPT ANY TWO QUESTIONS)

QUESTION TWO (20 MARKS)

- (a) Define a transcendental function (2 marks)
- (b) A function need not be specified by a formula. Justify the statement by giving two examples and representing them on a graph. (4 marks)
- (c) Give the fundamental definition of a derivative of a function $f(x)$. What is the physical significance of such a derivative (4 marks)
- (d) The normal temperature T of a healthy animal is 40.6°C . During an illness the temperature of the animal is expressed as $T(t) = 40.6 + 1.2t - 0.4t^2$, where t is in days
 - (i) Sketch a graph of the temperature T , against t for the first five days (2 marks)
 - (ii) From the graph, find the rate of change of temperature with time. (2 marks)
 - (iii) From the graph, explain when the animal started recovering from the illness? (2 marks)
- (e) Describe the four types of monotonic functions (4 marks)

QUESTION THREE (20 MARKS)

- (a) Define the trigonometric functions for real number x and find their limiting values (6 marks)
- (b) Show that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$, hence Find an expression for $\cos 2\alpha$ (5 marks)
- (c) Integrate $\int_0^{\frac{\pi}{3}} (\cos^2 x - \sin^2 x) dx$ (4 marks)
- (d) Given that $\sin\alpha = \frac{3}{5}$, $\cos\beta = \frac{15}{17}$ where α is obtuse and β acute, without using tables or calculators, find the exact value of $\cos(\alpha + \beta)$ (5 marks)

QUESTION FOUR (20 MARKS)

- (a) Define exponential and logarithmic functions hence show that logarithms are exponents. (5 marks)
- (b)
 - (i) Prove that $\log_a mn = \log_a m + \log_a n$ for $a \neq 0$, (4 marks)
 - (ii) The loudness, L of sound of intensity, I in bels is given by $L = \log I - \log I_0$ where I_0 is the minimum intensity detectable by the ear. Find the loudness in decibel of a siren whose intensity is $3.1 \times 10^6 I_0$. (4 marks)
- (c)
 - (i) Sketch the graph of $y = (2)^{-x}$ (2 marks)
 - (ii) Give five characteristics of the curve above. (5 marks)

QUESTION FIVE (20 MARKS)

- (a) A force $\mathbf{F} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ displaces a body through a distance $\mathbf{d} = \mathbf{i} - 3\mathbf{j} - \mathbf{k}$, find
- (i) the work done on the body by the force (3 marks)
 - (ii) the angle between $\mathbf{F}$ and $\mathbf{d}$ (3 marks)
- (b) If $Z_1 = 2 + 3i$ and $Z_2 = 3 - i$. Evaluate
- (i) $\frac{1}{Z_2 - Z_1}$ (4 marks)
 - (i) $\left| \frac{1}{Z_2 - Z_1} \right|$ (2 marks)
- (c) The probability density function over $[0, 3]$ is given by $f(x) = \frac{x^2}{9}$. Find the expected values $E(x)$ and $E(x^2)$ hence determine the standard deviation. (8 marks)