MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SCO 109: LINEAR ALGEBRA FOR COMPUTER SCIENCE

NSTRUCTIONS TO CANDIDATES

(a) **Answer ALL the questions in Section A and ANY TWO Questions in Section B**

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Show that $w = \{ (x, y, z) | x + y + z = 0) \}$ is a subspace of $\mathbb{R}^3$ (5 marks)

b) Find the cross product of the following vectors $(2,4,6)$ and $(-3,6,1)$ (3 marks)

c) Reduce the following matrixes to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$ (3 marks)

d) Solve the following system of equation by Gauss elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6$$ (5 marks)

e) Find the equation of the plane through (-1,2,3) and perpendicular to the planes $2x - 3y + 4z = 1$ and $3x - 5y - 5z = 16$ 5 marks)

f) Calculate the dot product of the vectors $\vec{u} = (2, 2, 3)$ and $\vec{v} = (-1, 1, 4)$. (3 marks)

g) Find the inverse of the following matrix

$$\begin{bmatrix} 3 & 2 & 2 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$ (4 marks)

h) Show that the vector (22,6,-16) is a linear combination of the (2,2,0) and (4,2,-2) (5 marks)

SECTION B

QUESTION TWO 20 MARKS

a) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, −1). (4 marks)

b) Determine if (1,2,3,1), (2,2,1,3) and (-1,2,7,3) is linearly dependent (5 marks)

c) Find the angle between $u = 2i$ +2j+2k and $v = i + j + k$ (3 marks)

d) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

(4 marks)

e) Find the minors, cofactors and adjoint of the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

(4marks)

QUESTION THREE 20 MARKS

a) Solve by crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18$$

(5 marks)

b) Determine the value of 'a' so that the following systems in unknown *x*, *y* and *z* has

i). No solution

ii). More than one solution

iii). Unique solution

$$x - 3z = -3$$
$$2x + ay - z = -2$$
$$x + 2y + az = 1$$

(6 marks)

c) Find the distance D between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (5 marks)

QUESTION FOUR 20 MARKS

a) Define a vector space (6 marks)

b) Show that W= $\{(x, y) \mid x = 2y\}$ is a subspace for R^2. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)

QUESTION FIVE 20 MARKS

a) Define a Linear mapping (2 marks)

b) Let $T: R^2 \rightarrow R^2$ be defined by $T(x, y) = \{x + y, x\}$. Find the kernel of T (5 marks)

c) Let $T: R^3 \rightarrow R^2$ be the linear mapping defined by
$T(x, y, z) = (x + 2y - z, y + z, x + y + z)$. Find the basis and dimension of

i). Image of T

ii). Kernel of T (8marks)

d) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent.
(5 marks)