



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCES)

SAC 404: INVESTMENT AND ASSET MANAGEMENT II

DATE: 9/3/2023

TIME: 2:00-4:00 P.M

INSTRUCTIONS TO CANDIDATES.

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Evaluate the discounted mean term of a bond redeemable at par in 10 year's time with annual coupons of 8% at interest rates of 5%, 10% and 15%. Hence sketch a graph of the discounted mean term as a function of the interest rate over the range 5% to 15%. What is the relationship between the three forms of market efficiency (6 marks)
- b) Consider a market in which the Capital Asset Pricing Model (CAPM) holds
- i) Write down the equation of the Security Market Line, defining all the notation you use (2 marks)
- ii) In this market, the risk-free rate of interest is 9.44% per annum. There are only two risky assets in the market with the following attributes.

Rate of return (per annum)				Variance/Covariance Matrix			
State	Probability	Asset 1	Asset 2			Asset 1	Asset 2
1	0.2	10.00%	11.00%		Asset 1	0.00142	0.00379
2	0.3	15.00%	30.00%		Asset 2	0.00379	0.01146
3	0.1	18.00%	25.00%				
4	0.4	20.00%	40.00%				

- iii) Determine the weight of each asset in the market portfolio to be consistent with $\beta_1 = 0.46$, $\beta_2 = 1.36$. (2 marks)
- iv) Calculate the Market Price of Risk. (2 marks)
- c) In a market where the assumptions of the Capital Asset Pricing Model hold, there are two risky assets with the following attributes:

<i>Security</i>	<i>A</i>	<i>B</i>
Expected return (p.a.)	20%	16%

- i) Determine the composition of the market portfolio with expected return 18% per annum. (3 marks)
- ii) Calculate the beta of each security under the assumption that the risk-free rate of interest is 10% per annum. (3 marks)
- d) In a market in which the Arbitrage Pricing Theory (APT) model holds, the expected return is given by:

$$E[R_i] = r_f + b_{i,1} \lambda_1 + b_{i,2} \lambda_2 + \dots$$

- i) Define all the terms in the equation. (3 marks)
- Assume the risk-free rate $r_f = 0.04$. Consider two well diversified portfolios P_i with the following features in a two factor model:

	<i>P1</i>	<i>P2</i>
$E[R_i]$	15.50%	11.95%
$b_{i,1}$	(a)	(b)
$b_{i,2}$	1.5	0.7

- ii) Determine the values (a) and (b) for $\lambda_1 = 0.05$ and $\lambda_2 = 0.06$. (3 marks)

QUESTION TWO (20 MARKS)

An investor has £100 and is considering investing in two different stocks. The prices of both stocks are assumed to follow the lognormal model with the parameters below.

<i>Stock</i>	<i>Current price</i>	<i>Drift μ</i>	<i>Volatility σ</i>
A	£5	5%	20%
B	£5	8%	30%

- i) Calculate the expected value at time 3 of £100 invested in:
 - a) Stock A (2 marks)
 - b) Stock B (2 marks)
- ii) Calculate the standard deviation at time 3 of £100 invested in:
 - a) Stock A (3 marks)
 - b) Stock B (3 marks)

The investor decides to invest £50 in each stock.

- iii) Calculate the expected value of the investor's portfolio at time 3. (2 marks)
- The correlation of the two stocks is 0.3**
- iv) Calculate the standard deviation of the value of the investor's portfolio at time 3. (3 marks)
- v) Comment on the expected return and standard deviation of the portfolio compared to investing the whole £100 in one stock. (5 marks)

QUESTION THREE (20 MARKS)

- a) State the main assumptions of mean-variance portfolio theory (5 marks)

Consider a mean-variance portfolio model with two securities, with respective returns S_A and S_B , where the expected return $E[S_B] = 0.25E[S_A]$ and the variance of return $V[S_B] = 0.25V[S_A]$.

Let the correlation between the returns on the two securities be ρ .

- b) Determine, in terms of $E[S_A]$, the expected return on the minimum variance portfolio if: (6 marks)
 - a) $\rho = 0$
 - b) $\rho = 1$
- c)
 - a) Calculate the variance of the return on the minimum variance portfolio for part (ii)(b). (5 marks)
 - b) Comment on the risk in this portfolio. (3 marks)

QUESTION FOUR (20 MARKS)

A fund must make payments of £50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% pa at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond.

QUESTION FIVE (20 MARKS)

- a) What are the betas of the market portfolio and the risk-free asset? Use these beta values to complete the derivation of the security market line equation (10 marks)
- b) Suppose there are only two portfolios A and B available to invest in. A is a portfolio of risky assets and B is a portfolio consisting of just one risk-free asset. Show that the efficient frontier must be a straight line. (10 marks)