MACHAKOS UNIVERSITY

University Examinations 2019/2020Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 300: WAVE THEORY

DATE: 11/12/2019 **TIME: 11.00-1.00 PM**

INSTRUCTIONS TO CANDIDATES

I. *This paper consists of **FIVE** questions in **FIVE** printed pages. Answer **Question One** (compulsory) and any **TWO questions** from the rest of the questions.*

II. *Cheating will lead to automatic disqualification.*

IMPORTANT CONSTANTS

Acceleration due to gravity, $g = 9.81$ N/kg.

QUESTION ONE (30 MARKS)

a) Briefly explain what you understand by the following terms,

i. Simple harmonic motion. (2 marks)

ii. Fraunhoffer diffraction (2 marks)

b) Two harmonic oscillations of frequency ω_0 having amplitude 1.0 cm and initial phases 0 and $\frac{\pi}{2}$, respectively, are superposed. Determine the amplitude and the phase of the resultant vibration. (3 marks)

c) The displacement of a simple harmonic oscillator is given as

$$\psi = \psi_o \cos(\omega t + \varphi),$$

show that acceleration is given by $\ddot{\psi} = -\omega^2 \psi$. (3 marks)

d) Figure 1.0 shows a non-symmetric on time axis periodic square wave of a rectifier circuit. Use the trigonometric Fourier series analysis approach to determine,

i. Magnitude of the DC component. (2 marks)

ii. First four Fourier coefficients. (6 marks)

iii. Write the Fourier series. (2 marks)

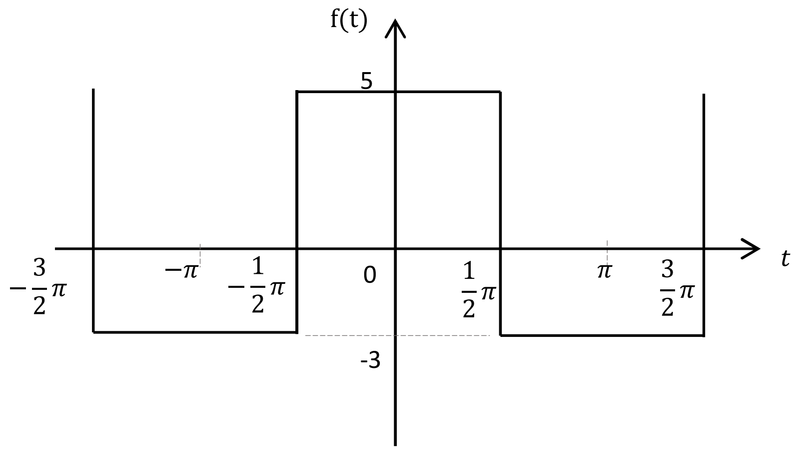

Fig. 1.0

e) A mass-spring system having mass m and springs constant k is executing simple harmonic motion about a fixed point. Derive the expression of the periodic time for the system.

(5 marks)

f) Use the method of complex number exponentials to show the superposition of harmonic waves:

$$a\cos \omega t + a\cos(\omega t + \phi) + a\cos(\omega t + 2\phi) + \cdots + a\cos(\omega t + [n-1]\phi)$$

$$= a\cos\left[\omega t + \frac{(n-1)}{2}\phi\right]\frac{\sin n\frac{\phi}{2}}{\sin\frac{\phi}{2}}$$

(5 marks)

QUESTION TWO (20 MARKS)

a) State the following theorem and principle

i. Fourier theorem (2 marks)

ii. Principle of superposition of waves. (2 marks)

b) Consider the following simple harmonic oscillations with angular frequency ω and phase angles ϕ_1 and ϕ_2

$$x_1 = a_1 \cos(\omega t + \phi_1)$$
$$x_2 = a_2 \cos(\omega t + \phi_2)$$

Use the principle of superposition to obtain the following expression of the amplitude for the resultant motion:

$$a = [{a_1}^2 + {a_2}^2 + 2a_1 a_2 \cos(\omega_1 - \omega_2)t]^{1/2}$$

Show that the resultant amplitude oscillates between the values $(a_1 + a_2)$ and $(a_1 - a_2)$. (8 marks)

c) Determine the Fourier series of the periodic triangular waveform shown in Figure 2.0. (8 marks)

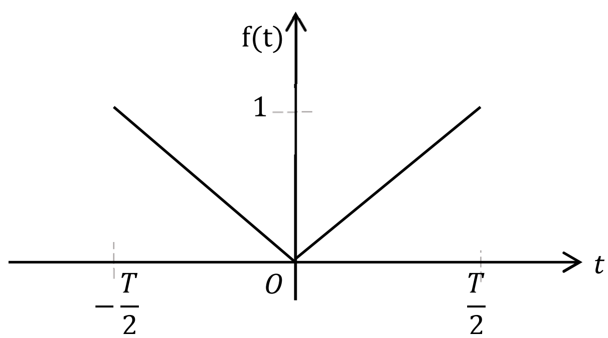

Fig. 2.0

QUESTION THREE (20 MARKS)

a) i) What is meant by a coupled oscillator? (2 marks)

ii) State two examples of a coupled oscillator. (2 marks)

b) A simple pendulum (mass M_p and length L_p is suspended from a cart (mass m_c) that can oscillate on the end of a spring of force constant k, as shown in the Fig 3.0.

i. Assuming that the angle φ remains small, write down the system's Lagrangian and the equations of motion for x and φ. (5 marks)

ii. Assuming that $m_c = M_p = L_p = g = 1$ and $k = 2$ (all in appropriate units), find the normal frequencies, and for each normal frequency find and describe the motion of the corresponding normal mode. (5 marks)

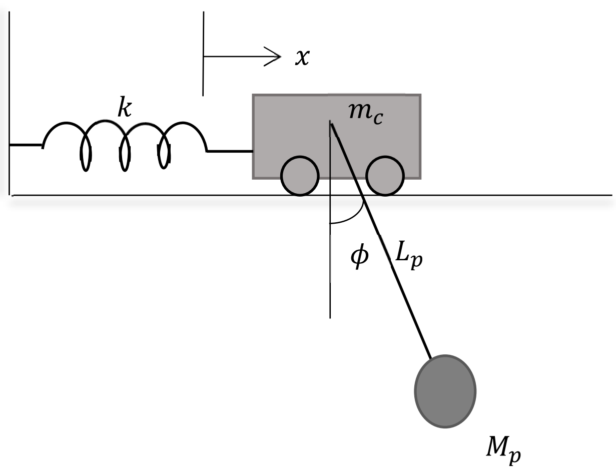

Fig. 3.0

c) As a model of a linear triatomic molecule (such as CO_2), consider the system shown in Fig 4.0 in which two identical atoms, each of mass m are connected by two identical springs to a single atom of mass M. To simplify matters, assume that the system is confined to move in one dimension.

i. Describe the normal modes qualitatively. How many are there? (3 marks)

ii. Write down the Lagrangian and find the normal frequencies of the system. (3 marks)

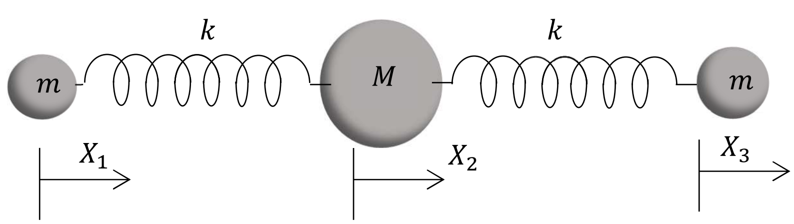

Fig. 4.0

QUESTION FOUR (20 MARKS)

a) (i) Describe two applications of the Newton's rings experiment. (2 marks)

(ii) Distinguish between diffraction and interference of light. (2 marks)

b) In a single slit diffraction experiment, the first minimum for red light 660 nm coincides with the first maximum of some other wavelength λ'. Find the value of λ'. (6 marks)

c) A set of newton's rings produced by monochromatic light are viewed through a traveling microscope. The diameter of the n^{th} and the $(n+5)^{th}$ dark rings are found to be 0.143 cm and 0.287 cm, respectively. Determine the wavelength of the light used? (10 marks)

QUESTION FIVE (20 MARKS)

a) (i) State four properties of diffraction through a single slit. (4 marks)

(ii) Distinguish between damped forced oscillation and undamped forced oscillation. (4 marks)

b) For a periodic function with a period of 2π such as the Fourier series

$$f(x) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} (A_n \cos \mathrm{nx} + B_n \sin \mathrm{nx})$$

Show that the amplitude A_o is given by $A_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$ (6 marks)

c) Two simple harmonic vibrations are represented by the following equations

$$x_1 = 3\sin\left(20\pi t + \frac{\pi}{6}\right)$$

$$x_2 = 4\sin\left(20\pi t + \frac{\pi}{3}\right).$$

Find the amplitude, phase constant and period of resultant vibration. (6 marks)