MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 107: ENGINEERING MATHEMATICS IV

DATE: 17/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS TO CANDIDATES

QUESTION ONE(COMPULSORY) (30 MARKS)

a) Find the derivative of $f(x) = 2x^2 - x + 5$ from the first principal. (3 marks)

b) Evaluate $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$ (3 marks)

c) Determine Taylors series expansion generated by the function $f\left(x\right) = \ln x$ upto the fifth term about $x = 2$. (4 marks)

c) Determine $\dfrac{dy}{dx}$ given that $f(t) = \dfrac{t^2}{\sqrt{t^2+4}}$ (5 marks)

e) Find the area between the curve $x^2 = y$ and the straight line $y = 2$. (5 marks)

f) Show that the improper integral $\int_1^\infty \frac{1}{x^2}dx$ is convergent. (5 marks)

g) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (5 marks)

QUESTION TWO (20 MARKS)

a) Evaluate $\int_{0}^{\frac{\pi}{2}} x \sin x dx$ (3 marks)

b) Determine the inflection point of $f(x) = x^3 - 6x^2 + 9x + 1$ (4 marks)

c) A particle moves along a straight line and iys displacement x metres from the origin, after t seconds is given by $x = 4.9t^2$. find in terms of t

i) The average velocity over the interval t to $(t + \partial t)$ (4 marks)

ii) The instantaneous velocity. (4 marks)

d) Calculate the maxima and minima values of the function $y = x^3 - 4x^2 - 3x + 2$

and distinguish them. (5 marks)

QUESTION THREE (20 MARKS)

a) Determine $\frac{dy}{dx}$ given that

i) $y = x^x$ (2 marks)

ii) $y = (x^2 + 3x)^7$ (3 marks)

b) Calculate the volume obtained by rotating the area under the curve $y = 1 + x$

between $x = 1$ and $x = 2$ about the $x-$axis. (4 marks)

c) A farmer has 60m of fencing wire which he can put against an existing fence to form a rectangular enclosure for animals. What is the maxima area it can enclose? (5 marks)

d) Evaluate $\int \frac{3x + 7}{(x-1)(x^2+1)} dx$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Find the derivative of $f(x) = \cos x$ using the first principal. (5 marks)

b) A projectile is fired straight upwards with a velocity of $400 m/s$; its distance

above the ground t seconds after being fired is given by $s(t) = -16t^2 + 400t$.

where $s(t)$ is the distance of the particle from the ground after it has been fired.

Calculate the maximum height achieved by the particle (5 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through $(1,0)$ and has a stationary point at $(1,0)$. Find the value of α, β and the other turning points and hence sketch the curve (10 marks)

QUESTION FIVE (20 MARKS)

a) A curve is given by the parametric equations $x = a\cos^3\theta$ and $y = a\sin^3\theta$. Show that $\frac{dy}{dx} = -\tan\theta$ also find the value of $\frac{d^2y}{dx^2}$ where $\theta = \frac{\pi}{4}$.

(10 marks)

b) A cylinder is to be constructed so that the sum of height and base radius is 6cm. denoting base radius by rcm and volume Vcm^3. Show that $V = \pi\left(6r^2 - r^3\right)$. Hence show that the value of r which makes V a maxima. (10 marks)