# MACHAKOS UNIVERSITY

**University Examinations for 2022/2023 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR**

**BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)**

**BACHELOR OF SCIENCE (CIVIL ENGINEERING)**

**BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)**

**ECU 105/ ECU 111: ENGINEERING MATHEMATICS IV/ALGEBRA FOR ENGINEERS**

**DATE:** **TIME:**

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**INSTRUCTIONS TO CANDIDATES.**

**Answer question ONE and any other TWO questions.**

**QUESTION ONE (COMPULSORY) (PULSORY)**

a) Express $\frac{7}{3}\sqrt[3]{\frac{5}{9}}$ as $\sqrt[3]{k}$ (3 marks)

b) Solve the equation $3^x.7^{2x+1} = 37$ (3 marks)

c) Factorize $x^2 - 2x + 1$. Hence find the values of $x$ for which $x^2 - 2x + 1 > 0$ . (4 marks)

d) The polynomial $f(x) = 2x^3 + Ax^2 + Bx - 3$ is exactly divisible by $(x-1)$ and has a remainder $+9$ when divided by $(x+2)$. Find the values of the constants $A$ and $B$. Hence solve the equation $f(x) = 0$. (7 marks)

e) Express the following product in polar form $(2 + 2\sqrt{3}i)(1 + i)$ (5 marks)

f) The coefficient of $x$ in the expansion of $(1+ax)^5$ is equal to the coefficient of $x^4$ in the expansion of $\left(9+\frac{x}{3}\right)^6$, calculate the value of $a$ . (5 marks)

g) Find the 4th root of $-81$ (3 marks)

## QUESTION TWO (20 MARKS)

a) Solve the equation $\frac{2x}{x+1} = \frac{2x-1}{x}$ (3 marks)

b) When divided by $(x-1)$ the polynomial $g(x) = ax^3 - 3x^2 + bx + 6$ leaves a remainder of $-6$ and when divided by $(x+2)$ it leaves a remainder of zero. Find the values of the constants $a$ and $b$. Hence solve the equation $ax^3 - 3x^2 + bx + 6 = 0$. (7 marks)

c) In the binomial expansion of $\left(1+\frac{x}{n}\right)^n$ in ascending powers of $x$, the coefficient of $x^2$ is $\frac{7}{16}$. Given that $n$ is a positive integer,

i. Find the value of $n$.

ii. Evaluate the coefficient of $x^3$ in the expansion. (5 marks)

d) Convert the following complex number in its standard form $e^{3ln3-i\frac{\pi}{2}}$ (5 marks)

## QUESTION THREE (20 MARKS)

a) Convert the radian measure $\frac{\pi^c}{10}$ in to degree measure and hence determine $\tan\frac{\pi^c}{10}$. (3 marks

b) With the help of trigonometric identitiessin $\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, determine $\sin\frac{\pi}{12}$ correct to three decimal places. (3 marks)

c) Solve the inequality $-4 < 5 - 3x \leq 17$ (3 marks)

d) Simplify the rational expression $\frac{x^2+3x+2}{x^2-x-2}$ (3 marks)

e) Rationalize and simplify $\frac{\sqrt{10}}{\sqrt{5}-2}$ (3 marks)

f) Sketch the graphs of the functions $\cosh x$ and $\sinh x$ (5 marks)

## QUESTION FOUR (20 MARKS)

a) State De Moivre's Theorem. (2 marks)

b) Determine the roots of the equation of the complex variable z given by $z^2 - 12z + 52 = 0$. Hence represent these roots $z_1$ and $z_2$ in the same Argand diagram. (8 marks)

c) Find the square roots of $-7+24i$ by equating the real and imaginary parts of $-7+24i = (x+yi)^2$ . (4 marks)

d) Find the principal value of the argument of the complex number $z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$. Hence write $z$ in polar form, then use this expression to determine $z^{\frac{1}{3}}$ and $z^4$. (6 marks)

## QUESTIONS FIVE (20 MARKS)

a) Without using a calculator evaluate

i) $\cos 390^0$ (2 marks)

ii) $\sin 405^0$ (2 marks)

b) Derive the identity $(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$ (5 marks)

c) Given the complex numbers $z_1 = 1 + i$ and $z_2 = 2 - i$ find $\frac{z_1}{z_2}$ and represent the results in a well labelled Argand diagram. (5 marks)

d) Distinguish between the concepts permutation and combination (2 marks)

e) Simplify completely $\left(\cos\frac{7}{3}\pi + i\sin\frac{7}{3}\pi\right)\left(\cos\frac{2}{3}\pi - i\sin\frac{2}{3}\pi\right)$ leaving your answer in Cartesian/rectangular form. (4 marks)