MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (mathematics)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 363 TESTS OF HYPOTHESIS I

DATE: 17/12/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 marks)

b) Outline the general procedure used for hypothesis testing. (4 marks)

c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 marks)

d) Let X be a binomial random variable. We wish to test the hypothesis $H_0: p = 0.8$ against $H_a: p = 0.6$. Let $\alpha = 0.03$ be fixed. Find β for $n = 20$. (4 marks)

e) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_0 > \theta_1$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (7 marks)

f) A new gene has been identified that makes carriers particularly susceptible to a particular degenerative disease. In a random sample of 250 adult males born in Kenya, 8 were found to be carriers of the disease. Test whether the proportion of adult males born in Kenya carrying the gene is less than 10%. (5 marks)

g) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 marks)

QUESTION TWO (20 MARKS)

a) The average blood pressure for a control group C of 10 patients was 77.0mmHg. The average blood pressure in a similar group T of 10 patients on a special diet was 75.0mmHg. Carry out a statistical test to assess whether patients on the special diet have lower blood pressure. Take $\sum_{i=1}^{10} C_i^2 = 59{,}420$ and $\sum_{i=1}^{10} T_i^2 = 56{,}390$, $\alpha = 0.05$. (5 marks)

b) Let $X_1, \dots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 marks)

c) If $X \geq 1$ is the critical region for testing $H_0: \theta = 2$ versus $H_1: \theta = 1$ on the basis of a single observation from the population

$$f(x, \theta) = \theta \exp(-\theta x), 0 \leq x \leq \infty$$

Determine the type I and type II errors. (8 marks)

QUESTION THREE (20 MARKS)

a) Let p be the probability that a coin will fall head in a single toss. In order to test the hypothesis $H_0: p = \frac{1}{2}$, the coin is tossed 5 times and the hypothesis H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error. If $H_a: p = \frac{3}{4}$, find the probability of Type II error. (6 marks)

b) Suppose the time to failure X for a piece of equipment is an exponential random variable with parameter λ. Given a random sample $X_1, \dots, X_n$ of lifetimes derive the most powerful test to test $H_0: \lambda = 0.01$ versus $H_a: \lambda = 0.04$. (4 marks)

c) Let $X_1, \dots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$ vs $H_a: \mu \neq \mu_0$ at level α. (10 marks)

QUESTION FOUR (20 MARKS)

a) State two asymptotic properties of the likelihood ratio tests? (2 marks)

b) The intelligence quotients (IQs) of 17 students from one area of a city showed a sample mean of 106 with a sample standard deviation of 10 whereas the IQs of 14 students from another area chosen independently showed a sample mean of 109 with a sample standard deviation of 7. Is there a significant difference between the IQs of the two groups at $\alpha = 0.02$? Assume that the population variances are equal. (8 marks)

c) Let $x_1, \dots, x_n$ be a random sample from a normal population $N(\mu, \sigma^2)$. Use the likelihood ratio test to obtain the best critical region of size α for testing $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 \neq \sigma_0^2$. (10 marks)

QUESTION FIVE (20 MARKS)

a) Explain briefly the procedure for applying Neyman-Pearson lemma. (2 marks)

b) A new diet and exercise program has been advertised as a remarkable way to reduce blood glucose levels in diabetic patients. Ten randomly selected diabetic patients are put on the program and the results after one month are as in the table below:

Before	268	225	252	192	307	228	246	298	231	185
After	106	186	223	110	203	101	211	176	194	203

Do the data provide sufficient evidence to support the claim that the new program reduces blood glucose level in diabetic patients? Take $\alpha = 0.05$. (8 marks)

c) Given the data in the table below, test the hypothesis $H_0: \rho = 0$ versus $H_a: \rho \neq 0$. Take $\alpha = 0.05$. (10 marks)

x	-1	0	2	-2	5	6	8	11	12	-3
y	-5	-4	2	-7	6	9	13	21	20	-9