# MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

Course

**Second Year Second Semester**

**ECU 202: ENGINEERING MATHEMAICS VI**

**Date: ……………………..** **Time: ………………**

***Answer question ONE and any other TWO questions***

**QUESTION 1: 30 MARKS (COMPULSORY)**

a) Define
   i. Differential equations
   ii. Order of a differential equation
   iii. Ordinary differential equation (3 marks)

b) Show that the function $y = cosx$ is a solution of the ordinary differential equation defined by $\frac{d^2y}{dx^2} + y = 0$ (3 marks)

c) Form a differential equation from the primitive defined by $y = Ax^2 + Bx$ (5 marks)

d) Solve the differential equation defined by $\frac{dy}{dx} = \frac{x^2-2}{y}$ (5 marks)

e) Find the orthogonal trajectories of the curves whose equation is $x^2 - y^2 = c$ (5 marks)

f) Solve the differential equation defined by $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ (5 marks)

g) Define the particular solution of $\frac{d^2y}{dx^2} + 4y = \cos 3x$ (4 marks)

**QUESTION 2 (20 MARKS)**

a) Find the orthogonal trajectories of a curve whose equation is $y = \frac{x}{c+x}$ (5 marks)

b) Form a differential equation from the primitive defined by $y = A\cos 2x + B\sin 2x$ (5 marks)

c) A substance cools from 100°C to 60°C in 10 seconds. Find the temperature of the substance after 40 seconds assuming that the room temperature is 20°C. Given that $\frac{dT}{dt}\alpha(T - T_0)$. With T being the temperature at time t and $T_0$ being the room temperature (10 marks)

**QUESTION 3 (20 MARKS)**

a) Find the particular solution of the ordinary differential equation given by

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x - 2$ (6 marks)

b) Find the solution of the differential equation defined by $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$

(4 marks)

c) Using the method of undetermined coefficients solve the differential equation defined by

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2$ (10 marks)

**QUESTION 4 (20 MARKS)**

a) solve the linear differential equation defined by

$\frac{dy}{dx} - \frac{y}{x} = x^2$ (6 marks)

b) Show that the solution of the differential equation defined by

$\cos y \sin x \, dx + \sin y \cos x \, dy = 0$ is $\cos x \cos y = A$ (7 marks)

c) Use Laplace transform to solve the differential equation defined by

$\frac{dy}{dx} - 5y = 0 \,;\, y(0) = 2$ (7 marks)

**QUESTIONS 5 (20 MARKS)**

a) Determine whether the differential equation below is exact and find the solution of the differential equation

$2xydx + (1 + x^2)dy = 0$ (7 marks)

b) Show that the solution of an homogeneous linear differential equation with roots that are real and equal is given by

$y = (Ax + B)e^{\alpha x}$ (4 marks)

c) Solve the following differential equation

$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - \frac{2}{x}y = \ln x$ (9 marks)