MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 461: OPERATION RESEARCH II

DATE: 20/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTION: *Answer question one and any other two Questions* *Time 2 hours*

QUESTION ONE (30 MARKS)

a) Distinguish the following terms in reference to queuing theorem

i. Microscopic and Macroscopic traffic flow models

ii. Space head way and time head way

iii. Jockey and Balking queuing customers

iv. Customer inter-arrival and inter-service time in queuing theorem (8 marks)

b) Sate the Birkoff- Newman theorem and hence write the doubly stochastic matrix H as a convex combination of permutation matrices

$$H = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.8 & 0.2 & 0 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

(6 marks)

c) A company has an annual demand of 16000 units of item L each costing sh. 200. The cost per order is sh. 1000 while the cost of capital is 24%. To purchase 10,000 or more units the price is sh.190. Justify why the management should buy with or without the discount.

(6 marks)

d) Given that packets arrive to a communication node with a single output link according to a Poisson Process. Give the Kendall notation for the following cases:

i. The packet lengths are exponentially distributed, the buffer capacity at the node is infinite

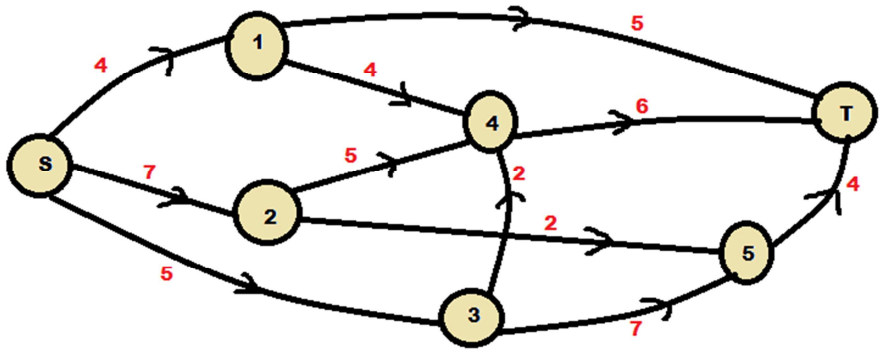
ii. The packet length is fixed, the buffer can store n packets and

iii. The packet length is L with probability p_L and l with probability p_l and there is no buffer in the node (8 marks)

e) Highlight four negative impacts of traffic jam (4 marks)

QUESTION TWO (20 MARKS)

a) Determine the maximum flow and the minimum cut in the following network (10 marks)



b) Using the project activities details recorded below

i. Draw the network to represent the project and mark the critical path

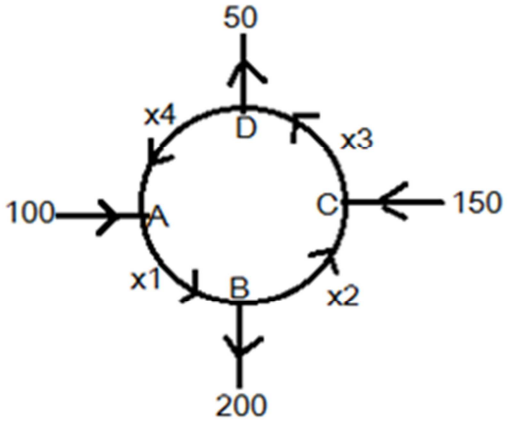
ii. Compute the total float for the non critical activities

iii. Determine the probability that the project is completed within 26 weeks (10 marks)

Activity	Preceding activity	Duration (weeks)	Standard deviation
A	----	5	1.5
B	----	6	2.0
C	A	7	2.5
D	A	4	1.0
E	A,B	2	0.5
F	C,D,E	4	1.5
G	D,E	10	3.0
H	F,G	3	1.5

QUESTION THREE (20 MARKS)

a) The figure below represents a roundabout (rotary). Assuming a one way flow and the numbers representing the number of vehicles per hour



i) Construct the mathematical model to represent the traffic flow

ii) Determine the minimum flow a long BC and the flows a long AB, CD and AD at that moment. (10 marks)

b) A company has 5 workers and 5 jobs. From the past experience workers can perform specific jobs as summarized below

Worker	Job performs
1	2
2	1,4
3	1,2
4	1,2,3,4
5	3,5

i. Represent the above information in a bipartite graph and a matrix

ii. Determine the maximal match (10 marks)

QUESTION FOUR (20 MARKS)

a) The following details relate to item K10 an input in a production process. Normal consumption is 100,000kg per week, maximum consumption is 150,000kg per week, minimum consumption is 50,000kg per week, re-order quantity is 600,000kg and re-order period is 4-6 weeks. Determine;

i. Re-order level

ii. Average stock level (4 marks)

b) In the brand switching challenge, the transition probability matrix is given by

$$\begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$$ If the market share vector at n^{th} share is $(0.4, 0.3, 0.3)$

i. What will be the market vector in $(n+1)^{th}$ stage

ii. Calculate the market share in the equilibrium situation. (8 marks)

c) There are five workers (x_i) who demand different pay for various five jobs (y_j) as summarized by matrix D.

$$D = \begin{pmatrix} \infty & 10 & 3 & 6 & 9 \\ 5 & \infty & 5 & 4 & 2 \\ 4 & 9 & \infty & 7 & 8 \\ 7 & 1 & 3 & \infty & 4 \\ 3 & 2 & 6 & 5 & \infty \end{pmatrix}$$

Determine the lowest-cost way to assign the jobs. (8 marks)

QUESTION FIVE (20 MARKS)

a) Highlight four conditions that may necessitate the use of simulation in modeling a system behavior (4 marks)

b) The Kulima limited company has designed a new pesticide whose production requires mainly two ingredients X1 and X_2. With the production function $y = \sqrt{x_1 x_2}$ where x_1 and x_2 are the units of ingredient 1 and 2 respectively required to produce one sachet of y. The price of each sachet and the cost of the ingredients are random variables such that

SP (y)	Probability
4000	0.15
4500	0.35
5000	0.20
5500	0.30

Cost (x1)	Probability
1000	0.1
1500	0.05
2000	0.35
2500	0.50

Cost (x2)	Probability
1500	0.20
2000	0.25
2500	0.15
3000	0.40

i. Given that the daily usage of x1 and x2 are set at 250 and 360 units respectively. Determine the expected profit of the company. (6 marks)

ii. Simulate the company's profit for 10 days using the random numbers; 58,71,96,30,24,18,09,24,18,46,23,34,27,85,13,99,24,44,49,79,49,74,16,32,23,02,56, 88,87,59. (10 marks)