



MACHAKOS UNIVERSITY

ISO 9001:2008 Certified 

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SST 103: LINEAR ALGEBRA

INSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Find the angle between the following vectors $4i + 4j + 6k$ and $i + 2j + 2k$ (4 marks)

b) Solve the homogenous system by Gauss elimination method

$$2x - 4y + 6z = 20$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 9z = 18 \quad (5 \text{ marks})$$

c) If $\begin{vmatrix} 4y & 20 \\ 4y & 8y + 4 \end{vmatrix} = 8$ find y (3 marks)

Calculate the cross product of the vectors $\vec{u} = (2, 2, 3)$ and $\vec{v} = (-1, 1, 3)$. (3 marks)

d) Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 2 \\ 1 & 2 & -1 \end{bmatrix} \quad (4 \text{ marks})$$

- e) Show that the vector $(2, -6, 3)$ is a linear combination of the $(3, -5, 4)$ and $(1, -2, -1)$
- f) Calculate the x and y values for the vector $(x, y, 1)$ that is orthogonal to the vectors $(3, 2, 0)$ and $(2, 1, -1)$. (4 marks)
- g) Determine if $(1, 2, 2, 1)$, $(2, 3, 4, 1)$ and $(3, 8, 7, 5)$ is linearly dependent (4 marks)
- h) Find the dot product of the following vectors $(3, 4, -1)$ and $(4, 6, 1)$ (3 marks)

SECTION B

QUESTION TWO 20 MARKS

- a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $T(x, y) = x + y$. Is T a linear map? (7 marks)
- b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T(x, y) = \{x + y, x\}$. Find the kernel of T (5 marks)
- c) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear mapping defined by
 $T(x, y, z) = (x + 2y - z, y + z, x + y + z)$. Find the basis and dimension of
 - i). Image of T
 - ii). Kernel of T (8 marks)

QUESTION THREE 20 MARKS

- a) Find the equation of the plane passing through the point $(3, -1, 7)$ and perpendicular to the vector $n = (4, 2, -5)$ (5 marks)
- b) Find the distance from the origin to the plane $2x + 3y - z = 2$ (5 marks)
- c) Find the parametric equations for the line of intersection of the plane
 $3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$ (5 marks)
- d) Find the distance D between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (5 marks)

QUESTION FOUR 20 MARKS

- a) Show that $w = \{(x, y, z) | x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$ (5 marks)
- b) Show that the vectors $u = (1, -1, 0)$, $v = (1, 3, -1)$ and $w = (5, 3, -2)$ are linearly dependent. (5 marks)
- c) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

(5 marks)

- d) Solve by crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18 \quad (5 \text{ marks})$$

QUESTION FIVE 20 MARKS

- a) Define a vector space (6 marks)
- b) Show that $W = \{(x, y) / x = 2y\}$ is a subspace for $\mathbb{R}^2$. (4 marks)
- c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)
- d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)