



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 432: PARTIAL DIFFERENTIAL EQUATION I

DATE: 29/3/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

a) Solve the differential equation $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$ (6 marks)

b) State the order of the partial differential equations below

i.) $\frac{\partial^2 z}{\partial x^2} = 1 + \left(\frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$ (3 marks)

ii.) $\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial^2 z}{\partial y^2}\right) = 2x\left(\frac{\partial z}{\partial x}\right)$ (3 marks)

c) State the degree of the following differential equations

i.) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{\frac{3}{4}}$ (4 marks)

ii.) $y\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial z}{\partial y}\right)^2 = 1 + \left(\frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$ (2 marks)

- d) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$ (6 marks)
- e) Solve the equation $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$ (6 marks)

QUESTION TWO (20 MARKS)

- a) Using Jacobi's method determine a complete integral of $p_1x_1 + p_2x_2 = p_3^2$ (10 marks)
- b) Consider the partial differential equation $p^2 + q^2 - 2pq \tanh 2y = \operatorname{sech}^2 2y$;
- i.) Write its Charpit's auxiliary equations (2 marks)
- ii.) Solve the p.d.e by considering its Charpit's auxiliary equations. (8 marks)

QUESTION THREE (20 MARKS)

Solve;

- a) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}$ (7 marks)
- b) $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2-y^2)}$ (8 marks)
- c) $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{(x^2+y^2)}$ (5 marks)

QUESTION FOUR (20 MARKS)

- a) Determine the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = c$ (8 marks)
- b) Solve the semi-linear equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$ (7 marks)
- c) Consider the partial differential equation $z = px + qy + p + q - pq$ with a complete integral of the form $z = ax + by + p + a + b - ab$; where a and b are arbitrary constants. Determine a general solution by finding the envelope of those planes that pass through the origin. (5 marks)

QUESTION FIVE (20 MARKS)

- a) Calculate the integral surface of the quasi-linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2 z)$ which contains the straight line $x + y = 0$, $z = 1$

(8 marks)

- b) in the one-dimensional unsteady flow of compressible fluid, the velocity u and c where

$c^2 = \frac{dp}{d\rho}$, p is pressure and ρ is density satisfying the following equations.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + 2c \frac{\partial c}{\partial x} = 0$$

$$2 \frac{\partial c}{\partial t} + 2u \frac{\partial c}{\partial x} + c \frac{\partial u}{\partial x} = 0$$

Prove that the characteristics are given by the differential equation $dx = (u \pm c)dt$ and that on the characteristic $dx = (u + c)dt$, $u + 2c$ is a constant. (12 marks)