MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (ACTUAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 334: FLUID MECHANICS II.

DATE: 15/12/2021 **TIME: 8.30-11.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the equation of a streamline passing through the point $p(2, 3)$ if velocity component for two dimensional flow are given by $u = 2$ and $v = 2$ (5 marks)

b) Given that the velocity potential for possible flow phenomena of irrotational flow is given by $\phi = 2x^2 y$. Determine the flow's streamline function φ (5 marks)

c) The velocity potential for a uniform stream flow $\phi(r,\ \theta,\ \varphi)$ has an expansion of the form $\phi(r,\ \theta,\ \varphi)=\sum_{n=0}^{\infty}\alpha_n r^n S_n(\theta,\ \varphi)$ with the series on the right being uniformly convergent with respect to r. Show that for a constant λ;

$$\frac{1}{r^{\lambda}}\int_0^r R^{\lambda-1}\phi(R,\ \theta,\ \varphi)dR=\sum_{n=0}^{\infty}\frac{\alpha_n r^n S_n(\theta,\ \varphi)}{n+\lambda}$$ (5 marks)

d) The complex potential for a fluid flow is given by $w(z)=v_0\left(z+\frac{a^2}{z}\right)$ where v_0 *and* a are positive constants. Determine;

i. The equipotential equation (5 marks)

ii. the velocity at any point of the flow (5 marks)

iii. The stagnation points (5 marks)

QUESTION TWO (20 MARKS)

a) Consider a two dimensional complex potential flow $w=z^2$. Determine

i. Stream function (3 marks)

ii. Velocity potential (3 marks)

iii. Sketch the flow's curves (3 marks)

b) Consider an irrotational flow in a cylindrical polar coordinates satisfying the Laplace equation $\frac{1}{R}\frac{d}{dR}\left(R\frac{\partial\phi}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial z^2}=0$. If the motion is $2-d$ and the coordinates are chosen that all physical quantities associated with the flow are independent of z, determine the flow's general solution representing its velocity potential function (11marks)

QUESTION THREE (20 MARKS)

a) Consider the flow over a stationary sphere with a uniform stream. Calculate the flow's

i. uniform stream velocity potential (3 marks)

ii. Perturbation velocity potential due to the stationary sphere (3 marks)

iii. General velocity potential function at infinity (6 marks)

b) Examine the flow given by $\phi=\frac{B}{2\pi}\theta$, where B is a constant (8 marks)

QUESTION FOUR (20 MARKS)

A $3-d$ doublet of strength μ whose axis is in the ox direction is distant a from the rigid plane $x = O$, which is the sole boundary of fluid of density ρ is infinite in extent.

a) Determine pressure at a point on the boundary distance r from the doublet given that the pressure at infinity is p_{∞} (10 marks)

b) Prove that the minimum pressure on the plane is at least a distance $\frac{a}{2}\sqrt{5}$ from the doublet. (10 marks)

QUESTION FIVE (20 MARKS)

Discuss a uniform flow past a fixed infinite circular cylinder of radius r *units*