MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 106: ENGINEERING MATHEMATICS III

DATE: 11/8/2021 **TIME: 11.00-1.00 PM**

INSTRUCTIONS:

Answer Question one (compulsory) any other two Questions

QUESTION ONE (30 MARKS)

a) Define a scalar and a vector quantity and give two examples to each (2 marks)

b) Solve for x, y and z given the following matrix equation.

$$\begin{bmatrix} x & 2 & -3 \\ 5 & y & 2 \\ 1 & -1 & z \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{bmatrix}$$ (3 marks)

c) If $a = 2i + 3j + 4k$, $b = 4i - j - k$. Evaluate

i. $|a + b|$ (2 marks)

ii. The angle between a and b (3 marks)

d) Given the vectors $a = 3i + 3j - 2k$, $b = -i + 3j + 4k$ and $c = 4i - 2j - 6k$.

Find a unit vector coplanar with a and b but perpendicular to c (5 marks)

e) Solve by Cramer's rule, the following system of equations

$$x+y+z=7$$
$$2x+3y+2z=17 \qquad \text{(5 marks)}$$
$$4x+9y+z=37$$

f) Find the eigen values of the matrix

$$A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$$ (5 marks)

g) Find an equation of the plane passing through the points $P=(-1,2,1)$, $Q=(0,-3,2)$, and $R(1,1,-4)$ (5 marks)

QUESTION TWO (20 MARKS)

a) The position vectors P, Q, R are respectively $P=i+j-k$, $Q=2k$ and $R=j-2k$. Find

$$\frac{|QP|}{|PR|}$$ (2 marks)

b) Find the area of a triangle whose vertices have the position vectors $a=i-j+2k$, $b=2j+k$ and $c=j+3k$ (4 marks)

c) If $A=\begin{bmatrix}2 & 3 & 4\\ 1 & 4 & 1\end{bmatrix}$; $B=\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 3\\ 4 & 1 & -1\end{bmatrix}$; $C=\begin{bmatrix}7 & 2\\ 1 & 0\\ -2 & 0\end{bmatrix}$ verify that $(AB)C=A(BC)$

(5 marks)

d) Solve the linear system using matrix method

$$3x-y+2z=13$$
$$2x+y-z=3 \qquad \text{(9 marks)}$$
$$x+3y-5z=-8$$

QUESTION THREE (20 MARKS)

a) If $a=2i-j+k$, $b=3i+j-k$, $c=i+\lambda j$, find the value of λ for which $a\cdot(b\times c)=-25$ (3 marks)

b) If $2A+B=\begin{bmatrix}2 & 3 & 1\\ 1 & 4 & 0\end{bmatrix}$ and $3A+2B=\begin{bmatrix}4 & 6 & 1\\ 2 & 3 & 5\end{bmatrix}$, find A and B (3 marks)

c) Find the equation of a line passing through the points $A = (1,0,-1)$ and $B = (-1,-1,0)$

(4 marks)

d) Find a unit vector which should lie on the plane determined by $a = 2i + j + k$, $b = i + 2j + k$ and perpendicular to $c = i + j + 2k$

(4 marks)

e) Solve the equations using Gauss's elimination method

$$\begin{aligned} x + y + z &= 6 \\ 3x + 3y + 4z &= 20 \\ 2x + y + 3z &= 13 \end{aligned}$$

(6 marks)

QUESTION FOUR (20 MARKS)

a) If $a = i + j + k$; $b = 2i - 3j + 2k$; $a = 2i - j + k$, $c = i - 2j - k$ find $(a \times b) \times c$

(3 marks)

b) Find parametric and symmetric equations of the line passing through the points $(1,-1,2)$ and $(2,1,5)$ (4 marks)

c) Find an equation of the plane passing through the point $P = (1,6,4)$ and containing the line defined by $R(t) = (1+2t, 2-3t, 3-t)$ (5 marks)

d) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(8 marks)

QUESTION FIVE (20 MARKS)

a) If $a = 2i + j + k$ and $b = i + j - 2k$, find the projection of a in the direction of b

(3 marks)

b) Given that matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, find

i. The eigen values and the corresponding eigen vectors (8 marks)

ii. Obtain $M = [x_1, x_2, x_3]$, hence M^{-1} (the inverse of M) (6 marks)

iii. Obtain AM and finally $M^{-1}AM$ (3 marks)