MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (INFORMATION TECHNLOLOGY)

SMA 437: BIOMATHEMATICS

DATE: 26/7/2017 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (30 MARKS)

a) Define the following terms as used in Biomathematics (5 marks)

   i. Biomathematics models
   ii. Carrying capacity
   iii. Empirical models
   iv. Deterministic models
   v. Stochastic models

b) A virus growing at a rate proportional to its size grows from 2000 to 3000 in 2hours. How long will it take for the virus to double? (6 marks)

c) State the law of decay and hence determine the percentage decay of a substance N when;

   i. time t=1/k
   ii. time t=2/k, for k=some constant (7 marks)

d) Given that C(t) is the concentration of a drug in the system at time t and C_0 is the concentration of the drug at time t=0. Suppose that constant doses C_0 are given every T units of time and that n=number of doses. Show that the concentration approaches equilibrium level as $n \to \infty$. i.e $C_\infty = {C_0}/{1 - e^{-kt}}$. (5 marks)

e) Discuss the assumptions of a simple deterministic model (4 marks)

f) Give an example of a disease epidemic that can be modeled by the following basic epidemiological models. (3 marks)

a) SI
b) SIR
c) SIRS

QUESTION TWO (20 MARKS)

Given the following general epidemic model equations;

$$dI(t)/dt = rI(t)S(t) - \alpha I(t)$$
$$dS(t)/dt = -rI(t)S(t)$$
$$dR(t)/dt = \alpha I(t)$$

With initial conditions; $S(0)=S_0>0$, $I(0)=I_0>0$ and $R(0)=0$.

a) Show that for the SI phase plane solution approach $I(t)=\rho \ln S(t)-S(t)-N-\rho \ln S_0$. Where $\rho=\alpha/r$. (10 marks)

b) Show that in an SIR model, when infections ultimately ceases spreading the number of susceptible $S(\infty)>0$ and total number of removals $R(\infty)=I_0+S_0-S(\infty)$. (10 marks)

QUESTION THREE (20 MARKS)

a) Given an SIR model with initial conditions $\{S(0), I(0), R(0)\}=(S_0, I_0, 0)$ and if $S_0>\rho$, such that $S_0-\rho=V$ and $N=\rho+r$. Show that if $V<<\rho$ then
   i. $R(\infty)\approx 2V$ (6 marks)
   ii. $S(\infty.)\approx \rho-V$ (4 marks)

b) Discuss any 5 reasons why modeling biological systems is important (5 marks)

c) Discuss any 5 criticisms to Malthusian doctrines. (5 marks)

QUESTION FOUR (20 MARKS)

a) Given a simple HIV model (SI model) with $\lambda(t)=\beta^{I(t)}/N(t)$ where; $I(t)/N(t)$ =proportion of individuals in the population who are HIV infected and β=probability of a sexual contact between a susceptible and infected individual.
   (i) Show that $I(t)=I_0 e^{(CB-V)t}$ (10 marks)
   (ii) Determine the doubling time of the number of infections, t_d where $1/v$=expected incubation period for an HIV infected individual. (5 marks)

b) State the Lotka-Volterra competition model equations and explain all the model parameters (5 marks)

QUESTION FIVE (20 MARKS)

a) Find the solution of the logistic model equation given by**;**
$^{dP}/_{dt}$=r(1-P/K)P, P(0)=P_0. Where r, k, and P_0 are some constants. (10 marks)

b) A population is observed to obey a logistic equation with eventual population of 20,000. If the initial population is 1,000, and 8 hours later, the observed population is 1,200. Find the reproductive rate and the time required for the population to reach 75% of its carrying capacity. (10 marks)