MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SST 402: BAYESIAN STATISTICS

DATE: 6/12/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the difference between the following terms

i. Loss function and Risk function. (5 marks)

ii. Conjugate prior and Jeffrey's prior. (5 marks)

b) A bask is going to invest in one of the Kenyan counties. The expected return depends on the economic situation (Improved, stable or worse) of the whole country. The payoff table is given below.

	Improved (0.5)	Stable (0.3)	Worse (0.2)
Machakos	50	-40	150
Kisumu	40	-20	100
Kitui	20	-15	40

i. What is the optimal decision under the maximin criteria? (3 marks)

ii. What is the decision under the maxi maxi criteria? (3 marks)

iii. What is the optimal decision under Bayesian role? (3 marks)

c) An engineer assesses the precision of a gauge by measuring the error of n instruments. She assumes that errors are independent realizations from normal distribution with mean zero and variance $1/\theta$. Obtain the posterior distribution of θ given that $\pi(\theta) = \text{gamma}(\alpha, \beta)$. (5 marks)

d) Suppose $X_1, X_2, \cdots, X_n$ is a random sample from $N(\theta, 1)$ and θ has prior distribution of $N(\alpha, 1)$

i. What is the posterior distribution for θ? (4 marks)

ii. Is the bayes estimator of (1) admissible? (2 marks)

QUESTION TWO (20 MARKS)

a) A random variable X has a distribution (Rayleigh) given by

$$f(x) = \begin{cases} 2\alpha x e^{-\alpha x^2}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

i. Compute $E(2X)$ and $Var(2X)$. (6 marks)

ii. Suppose that the prior for α is Gamma $(2\lambda, \beta)$ obtain bayes estimator under the squared loss function. Assume sample size n. (7 marks)

b) Consider the posterior distribution parameter θ

$$\pi(\theta|x) = 9\theta^{-10} \quad \text{if } \theta > 1.$$

Obtain the

i. Posterior mean for θ. (2 marks)

ii. Posterior median for θ. (2 marks)

iii. Posterior mode for θ. (3 marks)

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, \cdots, X_n$ be a random sample from density.

$$f(x|\theta) = \begin{cases} (1+x)^{-(\theta+1)}, & if\ x > 0, \quad \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

If θ has a gamma prior with parameters α and β.

i) Obtain the posterior distribution of θ. (3 marks)

ii) Obtain the Bayes estimator of θ. (3 marks)

iii) Show that the MLE estimator is not Bayes estimator. (3 marks)

iv) Find the Bayes estimator for θ under the loss function (4 marks)

$$L(\theta, a) = \frac{(\theta - a)^2}{\theta}$$

b) Three Marketing strategies are being considered for a new product. Potential demand for the product is unknown, but could be high, average, or low with probabilities 0.5, 0.3, and 0.2 respectively. Payoffs are given in the table below (thousands of dollars)

	Demand		
Strategy	High (0.5)	Average (0.3)	Low (0.2)
A	350	200	100
B	250	180	140
C	220	160	150

i. What is the optimal decision under minimax regret criteria? (3 marks)

ii. What is the optimal decision rule (using the probabilities)? (3 marks)

QUESTION FOUR (20 MARKS)

a) A random variable X has a Weibull distribution if its probability density is given $f(x) =$

$$\begin{cases} \frac{k}{2} x^{\beta-1} e^{-\alpha x \beta} & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

where $\alpha > 0,\ \beta > \theta$.

i. Express k in terms of α and β. (4 marks)

ii. Compute the variance of X. (4 marks)

b) Given the posterior $\pi(\theta) = 100\theta e^{-10\theta}$ if $\theta > 0$, and that $f(y|\theta) = \sqrt{\frac{\theta}{2\pi}}\, exp\left(-\theta \frac{y^2}{2}\right)$ if $-\infty < y < \infty$. Find $f\left(\theta|\underline{y}\right)$. (6 marks)

c) Suppose θ has a posterior distribution given as

$$f(x) = \begin{cases} 2\theta^{-3}, & \theta > 1 \\ 0, & \text{otherwise} \end{cases}$$

Find 95% highest posterior density (HPD) for θ. (6 marks)

QUESTION FIVE (20 MARKS)

a) The following amount from a lognormal distribution were paid on hospital liability policy

223, 125, 132, 141, 107, 133, 319, 126, 104, 145

Estimate μ and $\theta^2$by maximum likelihood. (7 marks)

b) Given the investment problem below with 3 decision alternatives and states of nature.

	State of nature	
Decision purchase	Good (0.7)	Poor (0.3)
Apartments (d_1)	\$50,000	\$30,000
Office (d_2)	\$100,000	\$40,000
Warehouse (d_3)	\$30,000	\$10,000

Suppose an investment analyst provides additional information on the economic condition (good $= g$, poor $= p$). This information may be negative (N) or positive (S)

Suppose $Pr\{S|g\} = 0.8$ $\Pr\{N|g\} = 0.2$

$Pr\{S|p\} = 0.1$ $\Pr\{N|p\} = 0.9$

Given the prior probabilities and conditional probabilities above

i. Compute

$Pr\{g|S\}$, $Pr\{p|S\}$, $Pr\{g|N\}$, $Pr\{p|N\}$, (6 marks)

ii. Draw the resulting decision free with additional information. (3 marks)

c) Briefly discuss the following Markov Chain Monte Carlo (MCMC) methods .

i. Gibbs sampling (2 marks)

ii. Metropolis Hastings (2 marks)