MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR BACHELOR OF ECONOMICS AND STATISTICS

SMA 361: THEORY OF ESTIMATION

DATE: 19/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

ANSWER QUESTION ONE AND OTHER TWO QUESTIOS

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Explain the meaning of the following terms as applied in theory of estimation

(i) Population

(ii) An estimator (4 marks)

(b) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 both unknown. Detemine the maximum likelihood estimator (MLE) of δ^2 . (6 marks)

(c) $Let\ x_1, x_2, .. x_n$ be a random sample drawn from a population with mean μ (unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (6 marks)

(d) Consider a normal distribution N(μ, δ^2).Let $\mu_1 = \bar{x}_n = \sum_1^n \frac{x_i}{n}$ and $\mu_2 = \bar{x}_m = \sum_1^m \frac{x_i}{m}$ where m>n, be estimators of μ. Determine which estimator is more efficient (7 marks)

(e) Given a population with a distribution whose pdf is

$$\left[f(x, \theta) = \frac{1}{\theta}, 0 \leq x \leq \theta, 0\ otherwise \right.$$

Using the method of moments obtain the estimator for θ. (7 marks)

QUESTION TWO (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\lambda+1)x^{\lambda}, & 0 < x < 1, \ \lambda > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of λ and test for unbiasedness (10 marks)

(b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased.

(10 marks)

QUESTION THREE (20 MARKS)

Three objects P, Q and R with weights Z_1, Z_2, and Z_3 respectively are weighed on a spring balance P and Q together weigh 75g, P and R weigh 45g and Q and R weigh 100g. Assuming the weights are independent with constant variance δ^2, find the least square estimates of Z_1, Z_2, and Z_3 (20 marks)

QUESTION FOUR (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf if $f(x, \theta)$ where θ is unknown. Let $T = t(x_1, x_2, \ldots x_n)$ be the necessary and sufficient condition for T to be the Minimum variance unbiased estimator (MVUE) of θ expressed in the form

$$\frac{\partial LogL}{\partial \theta} = \frac{T - \theta}{\lambda}$$

Show that T is MVUE of θ and $Var(T) = \lambda$ (10 marks)

(b) A random sample of size n is drawn from a normal population with $(0, \delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (10 marks)

QUESTION FIVE (20 MARKS)

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 95% confidence intervals for the true mean. (8 marks)

(b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)