



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 202: LINEAR ALGEBRA I

DATE: 24/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (**compulsory**) and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) Given that $\begin{pmatrix} x+y & 4s+2r \\ x-y & 2s-8r \end{pmatrix} = \begin{pmatrix} 16 & 22 \\ 12 & 14 \end{pmatrix}$ solve for x, y, s and r (4 marks)

b) Determine $10\mathbf{A} + 2\mathbf{B}$ Given that $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 7 \\ -2 & 4 \end{pmatrix}$ (2 marks)

c) Let $\mathbf{A} = \begin{pmatrix} 6 & 4 & 9 \\ 3 & -2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 5 & 1 \\ 1 & 7 & 1 \end{pmatrix}$

Compute;

i. $\mathbf{A}^2$ (3 marks)

ii. $\mathbf{AB}$ (3 marks)

d) Reduce the matrix

e) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 0 \\ -1 & -2 & -1 & 1 \\ 1 & -1 & 3 & 3 \end{pmatrix}$ to echelon form. Hence state the Rank of the matrix. (4 marks)

f) Compute the determinants of the following matrices;

i. $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (2 marks)

ii. $\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}$ (4 marks)

- iii. Given $A = \begin{pmatrix} \lambda & \lambda \\ 6 & \lambda - 4 \end{pmatrix}$ find λ if determinant of A is zero. (4 marks)
- g) Solve the equations below using elimination method;
- $$x_1 + x_2 + x_3 = 5$$
- $$x_1 + 2x_2 + 3x_3 = 10$$
- $$2x_1 + x_2 + x_3 = 6$$
- (4 marks)

QUESTION TWO (20 MARKS)

- a) Given that $Q = \begin{pmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{pmatrix}$ find the minor and co – factors of each element of Q (10 marks)
- b) Let $R = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix}$. Compute the adjoint of matrix R . (10 marks)

QUESTION THREE (20 MARKS)

- a) Solve the following system of equations using Gauss – Jordan elimination method.
- $$x_1 - x_2 + x_3 + 2x_4 = 1$$
- $$2x_1 - x_2 + 3x_4 = 0$$
- $$-x_1 + x_2 + x_3 + x_4 = -1$$
- $$x_2 + x_4 = 1$$
- (10 marks)
- b) Use Cramer's Rule to solve the system of equations below;
- $$2x + 3y - z = 1$$
- $$3x + 5y + 2z = 8$$
- $$x - 2y - 3z = -1$$
- (10 marks)

QUESTION FOUR (20 MARKS)

- a) Find the displacement vector from the point P with coordinates $(1 + 2\mathbf{i}, 1 - 2\mathbf{i})$ to the point Q with coordinates $(3 + \mathbf{i}, 2\mathbf{i})$ (3 marks)
- b) Find $\|\mathbf{y}\|$ if $\mathbf{y} = [1 \ 2 \ 3 \ 4]$ (3 marks)
- c) If $\mathbf{z} = [1 \ 0 \ -3 \ 2 \ -1]$ find its normalized vector. (4 marks)

- d) Find $\mathbf{x}$ if $3\mathbf{x} + 2\mathbf{y} = \mathbf{b}$ where $\mathbf{y} = \begin{bmatrix} 3 \\ 1 \\ 6 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 4 \\ 1 \end{bmatrix}$ (4 marks)
- e) Find the inverse of $\mathbf{B}$ i.e. $\mathbf{B}^{-1}$ given the matrix; $\mathbf{B} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix}$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Find the inverse of matrix $\mathbf{Q}$ given that; $\mathbf{Q} = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 2 & 0 \\ 3 & 1 & 1 \end{pmatrix}$ using row-reduction to echelon form of an augmented matrix (8 marks)
- b) Consider the following matrices;
 $\mathbf{S} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $\mathbf{T} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ find: $(\mathbf{S} + \mathbf{T})^2 - (\mathbf{S}^2 + 2\mathbf{ST} + \mathbf{T}^2)$ (8 marks)
- c) Show that the matrix equation

$$\begin{bmatrix} 1 & 1 & -2 \\ 2 & 5 & 3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \\ 5 \end{bmatrix}$$
is equivalent to the vector equation $u \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + v \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} + w \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \\ 5 \end{bmatrix}$ (4 marks)