



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 21/1/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS: Answer **QUESTION ONE** and any other **TWO** of the remaining questions.

QUESTION ONE: COMPUSARY (30 MARKS)

- a) Determine the values of a so that $\vec{A} = ai - 2j + k$ and $\vec{B} = 2ai + aj - 4k$ are orthogonal. (3 marks)
- b) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by
$$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$$
 (4 marks)
- c) If $\vec{A} = 3i - 2j + 4k$ and $\vec{B} = 2i - 3j - 2k$ determine the angle θ between the two vectors and the area of the triangle formed by vectors $\vec{A}$ and $\vec{B}$. (5 marks)
- d) Find $\nabla \circ (\nabla \phi)$ where $\phi = \phi(x, y, z)$ is a differentiable scalar function (5 marks)
- e) Given the space curve $x = 2t$, $y = t^2$ and $z = \frac{2}{3}t^3$. Find; Curvature k and Torsion τ (6 marks)
- f) Find the length of the arc of the curve $\vec{r} = \cos t i + e^t \sin t j + e^t k$ from $0 \leq t \leq \pi$ (7 marks)

QUESTION TWO (20 MARKS)

- a) A straight line passes through the points $A(2,3,4)$ and $B(5,1,6)$
- Write down the direction vector of $\overrightarrow{AB}$ (2 marks)
 - State the vector equation of the line AB (2 marks)
 - State the Cartesian equation of the line AB (2 marks)
 - State the parametric equation of the line AB (2 marks)
- b) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i + 6j + 7k$ (3 marks)
- c) If $\vec{A} = (y^2 - x^2z^2)\vec{i} + (x^2 + y^2)\vec{j} - x^2yz\vec{k}$, determine $\text{curl}\vec{A}$ at the point $(1,3,-2)$ (3 marks)
- d) Prove the vector triple product $\vec{A}(\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \circ \vec{C}) - \vec{C}(\vec{A} \circ \vec{B})$ (6 marks)

QUESTION THREE (20 MARKS)

- a) The vectors $\vec{a} = 2i - j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = 3i - pj + 5k$ are coplanar. Find constant p (3 marks)
- b) The equation of the two planes Π_1 and Π_2 are given by, $2x + 3y - 4z + 1 = 0$ and $3x - 5y + 6z + 8 = 0$ respectively. Calculate the angle between the two planes. (5 marks)
- c) Find the length of the arc of the curve $\vec{r} = \cos t \vec{i} + e^t \sin t \vec{j} + e^t \vec{k}$ from $0 \leq t \leq \pi$ (5 marks)
- d) Find the second fundamental form on the surface $\vec{r} = u\vec{i} + v\vec{j} + (u^2 - y^2)\vec{k}$ (7 marks)

QUESTION FOUR (20 MARKS)

- a) Determine the value of p so that $\vec{a} = 2i + 3j + k$ and $\vec{b} = pi - 2j - 2k$. (2 marks)
- b) Find the unit vector paralleled to the resultant of vectors $\vec{r}_1 = 2i + 4j - 5k$, and $\vec{r}_2 = i + 2j + 3k$ (3 marks)
- c) Determine the volume of the parallelepiped whose edges are given by $\vec{a} = 2i - 3j + 4k$, $\vec{b} = i + 2j - k$ and $\vec{c} = 3i - j + 2k$ (4 marks)
- d) Consider the surface $\vec{r} = u\vec{i} + v\vec{j} + (u^2 + v^2)\vec{k}$

- i. Find the equation to the tangent plane at the point P on the surface where $u = -1$ and $v = 1$ (5 marks)
- ii. Equation to the normal line at P (6 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\vec{r} = l \cos t \mathbf{i} + l \sin t \mathbf{j} + mt \mathbf{k}$ where l and m are constants determine the following

- a) The unit tangent $\vec{T}$ (3 marks)
- b) The curvature K and the radius of curvature ρ (5 marks)
- c) The unit normal $\vec{n}$ (3 marks)
- d) The unit binormal $\vec{b}$ (4 marks)
- e) The Torsion τ and the radius of the Torsion σ