



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 104: CALCULUS I

DATE: 27/9/2019

TIME: 2:00 – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question one and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate the following limits

i). $\lim_{x \rightarrow 1} (x^2 - 2x + 3)$ (2 marks)

ii). $\lim_{x \rightarrow 0} \frac{(x^2 + 9)}{(x - 2)^2}$ (2 marks)

b) Find the $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$ (3 marks)

c) Find the limit of $\lim_{\theta \rightarrow \infty} \frac{\sin 2\theta}{\theta}$ (3 marks)

d) Find the derivative of $f(x) = 2x^2 - x + 5$ from the first principal (3 marks)

e) Find the derivative of $f(x) = \sin x$ using the 1st principal (4 marks)

f) Obtain the derivative of $f(x) = 5x^2 - \cos x + 2$ (3 marks)

g) Show that $\sin^2 \theta + \cos^2 \theta = 1$ (5 marks)

h) Differentiate $\tan x$ using the first principle (5 marks)

QUESTION TWO 20 MARKS

a) Evaluate

$$\lim_{x \rightarrow 0} x \sin \left(\frac{\sqrt{x+2}}{x} \right) \quad (3 \text{ marks})$$

$$\lim_{x \rightarrow 2} \sqrt{\frac{5x^3 - 15}{x^3}} \quad (2 \text{ marks})$$

b) For each of the following functions obtain their derivatives from first principal

i) $f(x) = \frac{1}{2x+1}$ (3 marks)

ii) $f(x) = 2x + 3$ (2 marks)

c) Obtain the derivative of the following functions

i) $y = \sin^{-1} x$ (3 marks)

ii) $y = \sin^3 x \cos^2 x$ (3 marks)

d) Find $\frac{dy}{dx}$ for $x^3 + 3x^2y + y^3 = 1$ (4 marks)

QUESTION THREE 20 MARKS

a) Calculate the maxima and minima values of function $y = x^3 - 3x^2 + 2$ and distinguish between them and sketch the graph. (5 marks)

b) A cylinder is to be constructed so that the sum of height and ball radius is 6cm. Denoting ball radius by r cm, volume v cm³. Show that $r = \pi(6r^2 - r^3)$. Hence show the value of r which make V a maxima. (5 marks)

c) Differentiate the following functions

i) $y = \sin^{-1} x$

ii) $\tan^{-1} x$ (10 marks)

QUESTION FOUR 20 MARKS

- a) Differentiate the following functions with respect to y
- i) $(x + y)^3 = 2x - y$ (5 marks)
 - ii) $x^3 + x^3y^4 = 4y$ (5 marks)
 - iii) $x^3y + y^4\sqrt{x} + 4y = (x + y)^2$ (5 marks)
- b) Differentiate the following functions $f(x) = (e^{2x})^{2x} + \cos 2x + \cos^{-1} x$ (5 marks)

QUESTION FIVE 20 MARKS

- a) find $\frac{dy}{dx}$ for the following function
- i) $x = \sin^3 \theta \quad y = \tan^2 \theta$ (5 marks)
 - ii) $x = 3t^2 + 4t \quad y = e^{3t} + \cos^{-1} t$ (5 marks)
- b) Differentiate $4xy + (4x^4y^3 + 12xy^5 = 12 + 12y)$ with respect to x (5 marks)
- c) Obtain the derivative of the following functions
- $f(x) = 3(2x + 2)^4 + x^3$ (5 marks)