MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 231: DYNAMICS I

DATE: 24/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (30MARKS)

a) On turning a corner, a motorist rushing at $20m/s$ finds a child on the road $50m$ ahead. He instantly stops the engine and applies brakes so as to stop the car within $10m$ off the child. Determine;

i. The retardation (3 marks)

ii. Time required to stop the car (2 marks)

b) The torque $\vec{\Lambda}$ acting on a particle equals to time rate of change in angular momentum Ω. Show that;

$$\vec{\Lambda} = \frac{d\Omega}{dt}$$ (4 marks)

c) A plane has a rise of 5 *in* 12. A shot is projected with a velocity of $200m/s$ at an elevation of 30^0. Calculate the range of the plane if;

i.) The shot is fired up the plane (3 marks)

ii.) The shot is fired down the plane (2 marks)

d) A conservative force field defined by $\vec{F} = \left(y^2z^3 - 6xz^2\right)\hat{i} + 2xyz^3\,\hat{j} + \left(3xy^2z^2 - 6x^2z\right)\hat{k}$ has a scalar function defined by $\varphi = 3x^2z^2 - xy^2z^3$. Calculate the work done by the force field in moving a particle from the point $A\ (-2, 1, 3)$ to the point $B\ (1, -2, -1)$ (3 marks)

e) Derive the equation $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ for the simple harmonic oscillator (5 marks)

f) Two tanks sit side by side on the floor. The larger tank $(52kg)$ is to the left of the smaller tank $(34kg)$. A person pushes on the larger tank horizontally towards the right. The coefficient of static friction between the tanks and the floor is 0.35. Determine;

i.) The magnitude of the maximum force that the person can exert without moving any tank. (2 marks)

ii.) The force the larger tank exerts on the smaller tank. (2 marks)

g) A bomb is released from an aeroplane flying at $800km/hour$ at a height of $1000m$. Calculate the horizontal distance travelled by the bomb during the flight. (4 marks)

QUESTION TWO (20 MARKS)

a.) A particle of mass m travelling in a straight line is supplied with constant power P and is subjected to resistance Mkv^2, where v is its speed and k is a constant.

i.) Prove that the speed of the body cannot exceed a certain value. (6 marks)

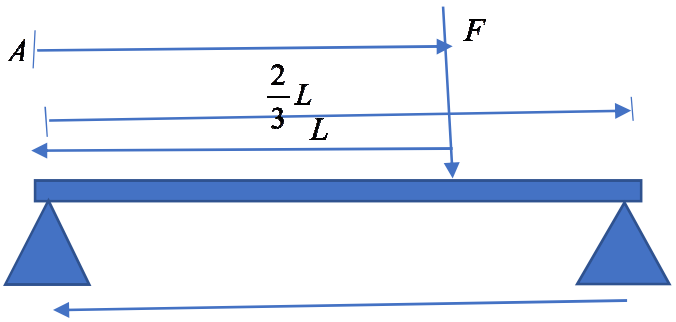
ii.) Prove that if it starts from rest it acquires half the maximum speed after travelling a distance $\frac{1}{3}k\ln\left(\frac{8}{7}\right)$. (7 marks)

iii.) If the power is then cut off and an additional retarding force of constant value $\vec{F}$ is imposed. Calculate the subsequent time which elapses before the body comes to rest. (7 marks)

QUESTION THREE (20MARKS)

a.) Three objects are arranged in a 2- dimensional coordinate system as; $A: 3kg\ (0.5m, 2.5m)$, $B: 3kg\ (3.5m, 2.5m), C: 4kg\ (2m, 0.5m)$ .Determine the center of mass of three objects. (6 marks)

b.) Consider a simple supported beam with a heavy load $6{,}000N$ as below.



Determine the forces at the ends of the beams. (7 marks)

c.) A particle of mass m moves along the $x-axis$ under the influence of a conservative force field having potential $\varphi(x)$. if the particle is located at the position x_1 and x_2 at the respective times t_1 and t_2. Prove that if E is the total energy then;

$$t_2 - t_1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E-\varphi(x)}} \qquad \text{(7 marks)}$$

QUESTION FOUR (20MARKS)

An object of mass $20kg$ moves with simple harmonic motion on the $x-axis$. Initially its located at the distance $4m$ away from the origin $(x=0)$, has a velocity of $15m/s$ and acceleration of $100m/s^2$ directed towards $x=0$.

a) Determine the position at any time t (7 marks)

b) Calculate;

i. Amplitude (1 mark)

ii. Period (2 marks)

iii. Frequency (2 marks)

c) Obtain the force on the object when $t = \frac{\pi}{10}\sec$ (8 marks)

QUESTION FIVE (20 MARKS)

A batman hits a cricket ball, giving it a velocity of $25m/s$ at an elevation of $\arctan\frac{4}{3}$. Calculate;

a) the greatest height attained (5 marks)

b) the range on level ground (5 marks)

c) the time of flight (5 marks)

d) height of the ball when it reaches the boundary which is $55m$ from the batsman. (5 marks)