Machakos University College

(A Constituent College of Kenyatta University)

UNIVERSITY EXAMINATIONS 2013/2014

SCHOOL OF COMPUTING AND APPLIED SCIENCES

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

ECU 107: ENGINEERING MATHEMATICS IV

DATE: 7/4/2014 **TIME:** 8.30 a.m. – 10.30 a.m.

Instructions:

Attempt question ONE and any other two questions.

Question 1

(a) Evaluate the following limits.

i. $\lim\limits_{x \to 0} \frac{x^2 - x - 2}{x^2 - 1}$ (3 marks)

ii. $\lim\limits_{x \to \infty} \frac{3x^2 - 6x}{4x - 8}$ (3 marks)

iii. $\lim\limits_{x \to \sigma} \frac{5x^2 - 3x + 2}{10x^2 - x + 100}$ (3 marks)

(b) Show that $\lim\limits_{x \to 1} \frac{2x^2 + 2x - 4}{x - 1} = 6$ (4 marks)

(c) Find the discontinuities/singularities of the function

$f(x) = \frac{x^2-3x+2}{x^2+5x-6}$ determine which discontinuities can be removed. (4 marks)

(d) Given $\lim_{h\to 0} \frac{\sin h}{h} = 1$ and $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$ show from first principles $\frac{d}{dx}(\sin x) = \cos x$ for all x (6 marks)

(e) Find $\frac{d}{dx}(x^2 \cos x)$ (3 marks)

(f) If *A, x* and *h* denote area, length of side and the altitude of a equilateral triangle respectively, then they are related by the formula

$A = \frac{\sqrt{3}}{4}x^2$ and $x = \frac{2\sqrt{3}}{3}h$ show that $\frac{dA}{dh} = 2$ (4 marks)

Question 2.

(a)Evaluate

$$\lim_{x\to 0} \frac{x \cos x}{x^2 + \cos x}$$ (2 marks)

(b) Suppose y is a differentiable function of x and satisfies $x^3 + y^3 = 2xy$.

Find $\frac{dy}{dx}$ (6marks)

(c) The distance s, of an object from a fixed point is given by

$s = 2t^3 - 2t^2 - 3t$, find:

i. The velocity
ii. The acceleration of the object at a time t.
iii. Show that the maximum distance attained from the fixed point is when $t = \frac{2+\sqrt{22}}{6}$ (6 marks)

(d) Water is poured into a conical paper cup at the rate of $\frac{2}{3}cm^3$ per second. If a cup is 6cm tall and the top of the cup has a radius of 2cm, how fast does the water level rise when the water is 4 cm deep?

Take volume of cone $=\frac{1}{3}\Pi r^2 h$ (6marks)

Question 3.

(a)Show that the $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$ (6 marks)

(b) Find the first three derivatives of $y = x \sin x$ (5 marks)

(c) Determine the stationary points of the function

$y = 4x^3 + 9x^2 - 12x + 3$ (9 marks)

Question 4.

(a) Find the singularities of the function

i. $f(x) = \frac{x-9}{x^2-81}$ (3marks)

ii. Which singularity is removable? (2marks)

iii. Redefine the function of $f(x)$ so that it becomes continuous at the removable singularity. (1mark)

(b) Given $y = \sin^{-1} x$, find $\frac{dy}{dx}$ (4marks)

(c) Given $y = \frac{e^{x^3+4x}}{e^x}$, find $\frac{dy}{dx}$ (4marks)

(d) Find the area A of the region between the parabola.

$x = \frac{1}{2}y^2$and the line $y = 2x - 2$ (6 marks)

Question 5

(a) Show that the function $f(x) = \frac{4x^3 - 16X^2 - 11x + 9}{2x-1}$ is continuous at $x = \frac{1}{2}$

(5 marks)

(b) From the definition of the derivative, show that if $f(x) = \frac{2x-3}{3x+4}$ then

$f' = \frac{17}{(3x+4)^2}$ (5 marks)

(c) Find the equation of the tangent and the normal to the curve given by the parametric equations $x = \frac{3t}{1+t}$ and $y = \frac{t^2}{1+t}$ at the point *t* =2

(d) Integrate $\int_0^1 (x-5)^4 dx$ (4marks)