MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 432: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 22/1/2021 **TIME: 2.0-4.00 PM**

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

QUESTION ONE (30MARKS) COMPULSORY

a) Under what conditions can the Lagrange equation $Pp+Qq=R$ be classified as: Linear, Semi-Linear and Quasi-Linear. (4 marks)

b) Find a partial differential equation arising from $z=ax^2+by^2+ab$ (4 marks)

c) Find the complete solution to $(y-z)\frac{\partial z}{\partial x}+(x-y)\frac{\partial z}{\partial y}=z-x$ (5 marks)

d) Find the integral curves of the equations:

i) $\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-(x+y+2z)}$ (6 marks)

ii) $y^2zp-x^2zq=x^2y$ (5 marks)

iii) $\frac{dx}{xz-y}=\frac{dy}{yz-x}=\frac{dz}{1-z^2}$ (6 marks)

QUESTION TWO (20 MARKS)

a) Show Pfaffian differential equation given by $z\left(x+y^2\right)dx+z\left(z+x^2\right)-xy\left(x+y\right)dz=0$ is integrable. (7 marks)

b) Find the solution of the equation $\left(x^2-y^2-z^2\right)p+2xyq=2xz$ (9 marks)

c) Eliminate the arbitrary constant from $z=ax^2-by^3+4ab$ (4 marks)

QUESTION THREE (20 MARKS)

a) Given that $u \equiv u(x, y, z) = c_1$, $v \equiv v(x, y, z) = c_2$ are solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Show that $F(u, v) = 0$ is a general solution of the Lagrange's equation. (10 marks)

b) Find the integral curves of the equation $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider a surface $F(x, y, z) = 0$ and let this surface intersect with the family of surfaces $G(x, y, z) = k$. Using Cramer's rule, derive the equations: $P' = RF_y - QF_z$, $Q' = PF_z - RF_x$, $R' = QF_x - PF_y$ (10 marks)

b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1$, z = 1 (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the general solution to the P.D.E.

$xp + yq = 3z$ (6 marks)

b) Eliminate the arbitrary function from $\phi\left(x+y+z\,,\, x^2+y^2+z^2\right) = 0$ (4 marks)

c) Find the integral surface of the equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x + y = 0$, $z = 1$ (10 marks)