MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE IN (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
SMA204: ALGEBRAIC STRUCTURES

DATE: 24/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer questions ONE and any other TWO questions.

Question One (30 Marks)

a) Define $\boldsymbol{f}: \mathfrak{R} \rightarrow \mathfrak{R}$ by $f(x) = \frac{1}{x-4}$. Determine the domain, range and $f^{-1}(x)$

(6 marks)

b) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(n) = n^2 + 2n + 1$ and $g(n) = 4n - 5$. Show that $(g \circ f)(n) \neq (f \circ g)(n)$ (4 marks)

c) Let $*$ be a binary operation defined on $\mathfrak{R}$ by $x * y = xy - x - y$. Show that $*$ is commutative but not associative.

(4 marks)

d) Let $*$ be a binary operation defined on a non-empty set A, with identity. Prove that the identity element is unique.

(4 marks)

e) Suppose $(G,*)$ is a group whose elements are $\{e, a, a^2, a^3, a^4, a^5\}$ with $a^6 = e$. Draw the Cayley table of G. Determine the inverse of each element and all subgroups of G.

(7 marks)

f) Differentiate between a zero divisor and a unit in a ring $(R, +, .)$. Determine all zero divisors and units in $(\mathbb{Z}_8, +, .)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Let G= $Z_5 \backslash \{0\}=\{1,2,3,4\}$ be the multiplicative group of integers modulo 5

(i) Find the multiplication table for G. (2 marks)

(ii) Find 2^{-1}, 3^{-1} and 4^{-1} (3 marks)

(iii) Find the cyclic subgroups of G generated by 2, 3 or 4 (3 marks)

(iv) Is G cyclic? Give reasons. (2 marks)

b) Find the order of each element in $\langle \mathbb{Z}_{12}, + \rangle$ by getting the subgroups it generates. Also state the inverse of every element (7 marks)

c) Prove that inverse of every element in a group is unique (3 marks)

QUESTION THREE (20 MARKS)

a) Explain what is meant by a commutative and an associative binary operation. (3 marks)

b) Prove that every cyclic group is Abelian (3 marks)

c) (i) Complete the table below so as to define a commutative binary operation $*$ on $S = \{a, b, c, d\}$

$*$	a	b	c	d
a	a	b	c	-
b	b	d	-	c
c	c	a	d	b
d	d	-	-	a

(4 marks)

(ii) Consider the table below

$*$	a_0	a_1	a_2	a_3
a_0	a_0	a_1	a_2	a_3
a_1	a_1	a_2	a_3	a_0
a_2	a_2	a_3	a_0	a_1
a_3	a_3	a_0	a_1	a_2

Solve for (I) $(x * a_3) * a_2 = a_0$ (2 marks)

(II) $a_3 x^2 a_1 = a_2$ (3 marks)

d) Let $\boldsymbol{f}: \mathbb{N} \rightarrow \mathfrak{R}$ be defined by

$$f(x) = \frac{1}{2x+3}$$

Is $f(x)$ a Bijection? (5 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms:

(i) a unit in a ring R (2 marks)

(ii) a zero divisor in a ring R (2 marks)

(iii) a nilpotent element in a ring R (2 marks)

(iv) an integral domain (2 marks)

(v) Commutative ring (2 marks)

b) The following tables show the Cayley table of a ring $< R, +, . >$

+	k	l	m	n	p	q	r
k	p	k	n	r	q	m	l
l	k	l	m	n	p	q	r
m	n	m	k	p	r	l	q
n	r	n	p	q	l	k	m
p	q	p	r	l	m	n	k
q	m	q	l	k	n	r	p
r	l	r	q	m	k	p	n

•	k	l	m	n	p	q	r
k	q	l	n	k	r	p	m
l	l	l	l	l	l	l	l
m	n	l	r	m	q	k	p
n	k	l	m	n	p	q	r
P	r	l	q	p	n	m	k
q	P	l	k	q	m	r	n
r	m	l	p	r	k	n	q

(i) Determine the additive and multiplicative identities. (2 marks)

(ii) Find the additive inverse of each element. (3 marks)

(iii) Determine the multiplicative inverse of each element. (3 marks)

(iv) Is there an annihilator in this ring? Give reason (2 marks)

QUESTION FIVE (20 MARKS)

a) What is a field? (2 marks)

b) Show that every field F is an integral domain (5 marks)

c) Show that every finite integral domain is a field (5 marks)

d) Let $G = D_4 = \{e, \alpha,\ \alpha^2,\ \alpha^3,\ \beta,\ \alpha^2\beta,\ \alpha^3\beta\}$ be a group and $H = \{e, \beta\}$ be a subgroup of G. Find all the left and right cosets of H in G (6 marks)

e) State the Lagrange's theorem (2 marks)