MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

SCO 109: LINEAR ALGEBRA FOR COMPUTER SCIENCE

DATE: 22/6/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Find the cross product of the following vectors $(2,4,-1)$ and $(3,6,1)$ (3 marks)

b) Reduce the following matrixes to be reduced echelon form

$$A=\begin{pmatrix}3 & 4 & -1 & 1\\ 1 & -1 & 3 & 2\\ 4 & -3 & 11 & 2\end{pmatrix}$$ (3 marks)

c) Solve the following simultaneous equation using inverse matrix method

$2x-y=4$

$3x+2y=6$ (3 marks)

d) If $\begin{vmatrix} y & 5 \\ y & 2y+1 \end{vmatrix}$=4 find y (3 marks)

e) Calculate the cross product of the vectors $\vec{u}$ = (1, 2, 3) and $\vec{v}$ = (−1, 1, 2). (3 marks)

f) Find the inverse of the following matrix

$$\begin{bmatrix}0 & 2 & 4\\ 2 & 4 & 2\\ 3 & 3 & 1\end{bmatrix}$$ (4 marks)

g) Show that the vector (11,3,-8) is a linear combination of the (1,1,0) and (2,1,-1)

h) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, −1). (4 marks)

i) Determine if (1,2,3,1), (2,2,1,3) and (-1,2,7,3) is linearly dependent (4 marks)

j) Find the angle between $u = 2i$ +2j+2k and $v = i + j + k$ (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Show that $(A-B)(A+B) = A^2 - B^2$ iff $AB = BA$ i.e A & B commute (4 marks)

b) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, show that $(AB)^T = B^T A^T$. (3 marks)

c) Determine the values of x for which the determinant is zero where

$$\begin{pmatrix} x-2 & 4 & 3 \\ 1 & x+1 & -2 \\ 0 & 0 & x-4 \end{pmatrix}$$

(4 marks)

d) Reduce $\begin{bmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{bmatrix}$ to reduced row-echelon (canonical) form. (3 marks)

e) Calculate the value of k for the vectors $\vec{u}$ = (1, k) and $\vec{v}$ = (–4, k) knowing that they are orthogonal. (3 marks)

f) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent. (3 marks)

QUESTION THREE (20 MARKS)

a) Let $a = 3i - j + k$ and $b = i + 2j - k$

i. Find $\vec{a} \times \vec{b}$

ii. Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$

iii. Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{b}$ (6 marks)

b) Let $A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix}$

i. Find det A

ii. Find Adj (A)

iii. Hence find A^{-1} (6 marks)

c) Find the cross product of the following vectors $(2,4,-1)$ and $(3,6,1)$ (4 marks)

d) Reduce the following matrixes to be reduced echelon form

$$A=\begin{pmatrix}3 & 4 & -1 & 1\\ 1 & -1 & 3 & 2\\ 4 & -3 & 11 & 2\end{pmatrix}$$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Solve using Crammer's rule.

$$x_1+3x_2+2x_3=3$$
$$2x_1+4x_2+2x_3=8$$
$$x_1+2x_2-x_3=10$$ (5 marks)

b) Solve the following equations using inverse matrix method

$$x+2y+z=4$$
$$3x-4y-2z=2$$
$$5x+3y+5z=-1$$ (5 marks)

c) Solve for x, y and z using Gauss-Jordan method

$$x+y+z=5$$
$$x+2y+3z=10$$
$$2x+y+z=6$$ (5 marks)

d) Solve by elimination method

$$x_1+x_2+x_3=5 \quad \cdots$$
$$x_1+2x_2+3x_3=10\cdots$$
$$2x_1+x_2+x_3=0\cdots$$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Define a vector space (6 marks)

b) Show that W= $\{(x,y)\ /\ x=2y\}$ is a subspace for R^2. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2,3,-4)$ and parallel to the vector $(3,5,-6)$ (5 marks)