MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SAC 203: ACTUARIAL MATHEMATICS I

DATE: 15TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE (30 MARKS)

a) Define the following lifetable notations.

i. ${}_nq_x$ **(2 marks)**

ii. L_x **(2 marks)**

b) Using ELT 15 (MALES) mortality, find the curtate expectation of life for, a 70-year-old pensioner. **(3 marks)**

c) The force of interest $\delta(t)$ is:

$$\delta(t) = 0.005t + 0.0001t^2 \text{ For all t.}$$

i. At $t = 8$,calculate the accumulated value of an investment of £ 100 made at time$t = 0$. **(5 marks)**

ii. Calculate the constant annual effective rate of interest over the eight year period. **(3 marks)**

d) Using the PMA92C20 table for both lives calculate:

$$2q^{1}_{65:65}$$

(5 marks)

e) If μ_x takes the constant value 0.025, at all ages, calculate the age x for which ${}_xp_0 = 0.5$ .what does this age represent? **(5 marks)**

f) For a force of mortality μ_x that is known to follow Gompertz' law ,calculate the parameters B and C if $\mu_{50} = 0.017609$ and $\mu_{55} = 0.028359$ **(5 marks)**

QUESTION TWO (20 MARKS)

a) For the first 5 years after arrival in a certain country, lives are subject to a constant force of mortality of 0.005. Thereafter lives are subject to mortality according to ELT 15(Males) with an addition of 0.039221 to the force of mortality.

i) A life aged exactly 30 has just arrived in the country.

- Show that the probability that the life will survive to age 35 is 0.97531. **(2 marks)**
- Find the probability that the life will survive to age 60. **(2 marks)**

ii) What is the probability that a life aged exactly 33 who has been in the country for 3 years will die between ages 50 and 51? (Assume that these lives will remain in the given country. **(4 marks)**

b) Suppose that Gompertz law $\mu_x = Bc^x$, holds for all $x \geq \alpha, c$ being greater than 1. Assume that $\mu_\alpha < \log c$.

i) Give a formula for$s(x)$. **(4 marks)**

ii) Show that $l_x\mu_x$ attains a maximum when $\mu_x = \log c$, and has no other stationary points. **(8 marks)**

QUESTION THREE (20 MARKS)

The force of interest $\delta(t)$ is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \leq 5 \\ 0.008t & 5 < t \leq 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

a) Calculate the present value of a unit sum of money due at time $t = 12$. **(13 marks)**

b) Calculate the effective annual rate of interest over the 12 years. **(2 marks)**

c) Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$. **(5 marks)**

QUESTION FOUR (20 MARKS)

a) A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract. **(8 marks)**

Basis:

Mortality:	AM92 Select
Interest:	4% per annum

b) An assurance contract provides a death benefit of £1,000 payable immediately on death, with a savings benefit of £500 payable on every fifth anniversary of the inception of the policy.

The following basis is used.

Force of mortality:	$\mu_x = 0.05$ for all x
Force of interest:	$\delta = 0.04$
Expenses:	None

Calculate the level premium payable annually in advance for life. **(12 marks)**

QUESTION FIVE (20 MARKS)

T_x denotes the future lifetime of a life aged x .

a) Write down the probability density function of T_x . **(1 mark)**

b) Using your answer to (a), show that **(10 marks)**

$$\frac{\partial}{\partial s}\log {}_sp_x = -\mu_{x+s} \text{ and } {}_tp_x = \exp\{-\int_0^t \mu_{x+s}\, ds\}$$

c) In a certain population, the force of mortality is given by,

	μ_x
$60 < x \leq 70$	0.01
$70 < x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. **(9 marks)**