MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF SCIENCE IN MATHEMATICAL MODELING AND COMPUTATION

SMA 802: MATHEMATICS METHODS FOR DIFFERENTIAL EQUATIONS

DATE: 21/6/2021 **TIME: 2.00-5.00 PM**

INSTRUCTIONS:
Answer question one and any other two questions.

QUESTION ONE (30 MARKS)

a) Define the chebyshev's polynomial of the 1st kind, $T_n(x)$ and 2nd kind $U_n(x)$ (3 marks)

b) Express $2x^3 + 3x^2 - x - 2$ in terms of Legendre polynomials. (4 marks)

c) Using Picard's method, solve the I.V.P $\frac{dy}{dx} = 2y - 2x^2 - 3$ $\quad y = 2 \quad at \quad x = 0$. Perform two iterations. (5 marks)

d) Prove that $p_n(x) = (-1)^n p_n(x)$ (4 marks)

e) Show that $f(x) = xy^2$ satisfies the Lipschitzs condition on the rectangle $\Re: |x| \le 1, \ |y| \le 1$ but doesn't satisfy a Lipschitzs condition on the strip $S: |x| \le 1, \ |y| \le \infty$ (5 marks)

f) Use the Laplace transformation to solve the equation;

$$u(x) = x + \int_0^x u(y)\cos(x - y)dy$$ (5 marks)

g) State and prove uniqueness theorem (4 marks)

QUESTION TWO (15 MARKS)

Show that;

a) $\left[1-2xz+z^2\right]^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n p_n(x), \quad |x| \le 1, \quad |z| < 1$ (10 marks)

b) $\left[1-2xz+z^2\right]^{-\frac{1}{2}}$ is a solution of the equation $z\dfrac{\partial^2(zv)}{\partial z^2} + \dfrac{\partial}{\partial}\left[(1-x^2)\dfrac{\partial v}{\partial x}\right] = 0$ (10 marks)

QUESTION THREE (15 MARKS)

a) Consider the I.V.P $\dfrac{dy}{dx} = e^x + y^2; \quad y(0) = 1$. Use the Picard's method of successive approximation to determine a sequence of two functions which approach its solution.

(8 marks)

b) Determine the third approximation of the solution of the equation $\dfrac{dz}{dx} = x^3(y-z); \quad \dfrac{dy}{dx} = z$ by Picard's method where $y = 1, \quad z = \dfrac{1}{2}$ where $x = 0$

(12 marks)

QUESTION FOUR (15 MARKS)

Prove that ;

a) $\dfrac{1-t^2}{1-2tx-t^2} = T_0(x) + 2\sum_{n=1}^{\infty} T_n(x)t^n$ (10 marks)

b) $\sum_{r=0}^{n} T_{2r}(x) = \dfrac{1}{2}\left[1 + \dfrac{1}{(1-x^2)^{\frac{1}{2}}} U_{2n+1}(x)\right]$ (10 marks)

QUESTION FIVE (15 MARKS)

Consider the Bessel equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$$

a) Determine its solution (10 marks)

b) define the Bessel's function of the first kind (5 marks)