MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE IN (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 23/7/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer questions ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find a unit vector normal to the plane S containing $P(0,1,1)$, $Q(1,0,-1)$, $R(1,-1,0)$ (3 marks)

b) Show that $\left||a|-|b|\right| \le |a \pm b|$ for all a and b (4 marks)

c) Show that $x = te_1 + (t^2+1)e_2 + (t-1)e_3$ is a regular parametric representation for all t and find projections onto the x_1x_2 and x_1x_3 planes (4 marks)

d) Find the equation of the line tangent to the curve generated by $x = te_1 + t^2e_2 + t^3e_3$ at $t = 1$ (4 marks)

e) Compute the length of the arc $x = 3(\cosh 2t)e_1 + 3(\sinh 2t)e_2 + 6te_3$, $0 \le t \le \pi$ (4 marks)

f) Let $f(t) = (1+t^3)e_1 + (2t - t^2)e_2 + te_3$, $g(t) = (1+t^2)e_1 + t^3e_2$. Find:

   a) $f(a) \bullet g(b)$ (2 marks)

   b) $f(t) \times g(t)$ (2 marks)

g) Find the curvature vector k and curvature $|\kappa|$ on the curve $x = te_1 + \frac{1}{2}t^2e_2 + \frac{1}{3}t^3e_3$ at the point $t = 1$ (7 marks)

QUESTION TWO (20 MARKS)

i. Using the definition of the limits, show that

$$\lim_{t \to 1}\left(t^2e_1 + (t+1)e_2\right) = e_1 + 2e_2$$ (4 marks)

ii. Let $u = (\sin t)e_1 + 2t^2 + te_3$, $(t > 0)$, and $t = \log\theta$, find $\dfrac{du}{d\theta}$ as a:

a) Function of θ (3 marks)

b) Function of t (3 marks)

iii. Let $a = e_1 - 2e_2 + 3e_3$ and $b = 2e_1 - 3e_2 + e_3$.

a) Show that b is in $S_3(a)$ (2 marks)

b) Find a $\delta > 0$ such that $S_\delta(b)$ is contained in $S_3(a)$ (4 marks)

c) Find ε_1 and ε_2 such that $S_{\varepsilon_1}(a)$ and $S_{\varepsilon_2}(b)$ are disjoint (4 marks)

QUESTION THREE (20 MARKS)

i. Evaluate $\lim\limits_{t\to 2}\left[(3t^2+1)e_1 - t^3e_2 + e_3\right]$ (3 marks)

ii. Show that the curve generated by;

$x = (-1 + \sin 2t\cos 3t)e_1 + (2 + \sin 2t \sin 3t)e_2 + (-3 + \cos 2t)e_3$

Lies on the sphere of radius 1 about $a = -e_1 + 2e_2 - 3e_3$ (4 marks)

iii. Find the equation of the tangent line and normal to the curve $x = (1+t)e_1 - t^2e_2 + (1+t^3)e_3$ at $t = 1$ (5 marks)

iv. If $u = (3t^2+1)e_1 + (\sin t)e_2$ and $v = (\cos t)e_1 + e^te_3$. Find:

a) $\dfrac{d}{dt}(u \bullet v)$ (3 marks)

b) $\dfrac{d}{dt}(u \times v)$ (3 marks)

QUESTION FOUR (20 MARKS)

i. Let $f(t) = (\sin t)e_1 + te_3$ and $g(t) = (t^2+1)e_1 + e^te_2$. Find:

a) $\lim\limits_{t\to 0}\left(f(t) \bullet g(t)\right)$ (2 marks)

b) $\lim\limits_{t\to 0}\left(f(t) \times g(t)\right)$ (2 marks)

ii. Show that the representation $x_1 = (1+\cos\theta)$, $x_2 = \sin\theta$, $x_3 = 2\sin(\theta/2)$, $-2\pi \le \theta \le 2\pi$ is a regular and lies on then sphere of radius 2 about the origin and the cylinder $(x_1 - 1)^2 + x_2^2 = 1$ (4 marks)

iii. Prove the *Cauchy-Schwartz* inequality, $|a \bullet b| \le |a||b|$, with equality holding if and only if a and b are linearly dependent (4 marks)

iv. Find the arc length as a function of θ along the epicycloid: $x_1 = (r_0 + r)\cos\theta - r\cos\left(\frac{r_0 + r}{r}\theta\right)$,

$x_2 = (r_0 + r)\sin\theta - r\sin\left(\frac{r_0 + r}{r}\theta\right)$ (8 marks)

QUESTION FIVE (20 MARKS)

i. Let $u = a(\cos t)e_1 + a(\sin t)e_2 + bte_3$, $a, b \neq 0$. Find:

a) $\frac{d^2u}{dt^2}$ (2 marks)

b) $\left|\frac{d^2u}{dt^2}\right|$ (2 marks)

ii. Let the curve be $x = (3t - t^3)e_1 + 3t^2e_2 + (3t + t^3)e_3$

a) Find a continuous unit principal normal and unit binormal along the curve (10 marks)

b) Find the curvature and torsion (6 marks)