



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS 1

DATE: 29/11/2019

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

1. The paper consists of **two** sections.
2. Section **A** is **compulsory** (30 marks).
3. Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (30 MARKS)

- a) What is the smallest possible uncertainty in the position of an electron moving with velocity 10^6 m/s? (3 marks)
- b) Show that $[x, E_x] = -i\hbar v$ (5 marks)
- c) Show that for a one-dimensional square - integrable wave-packet,

$$\int_{-\infty}^{\infty} j(x) dx = \frac{\langle p \rangle}{m}$$

Where $j(x)$, is the probability current? (7 marks)

- d) State the mathematical operator for position, momentum and energy for a normalised wave function (3 marks)
- e) Write down expression for time independent Schrodinger equation in one dimension. (2 marks)
- f) A particle of mass m is trapped in a one dimensional box with a potential described by

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{Otherwise} \end{cases}$$

Solve the Schrödinger equation for this potential. Taking $\Psi(x, t) = \Psi(x)e^{-iEt/\hbar}$ (8 marks)

- g) Explain two failures of classical mechanics from the photoelectric experiment that led to quantum mechanics approach (2 marks)

QUESTION TWO (20 MARKS)

Prove that $\frac{d\langle x \rangle}{dt} = \frac{\langle p \rangle}{m}$ Where $\langle x \rangle$ and $\langle p \rangle$ are the mean values of the coordinate and momentum of the particle, respectively.

QUESTION THREE (20 MARKS)

- a) State the mathematical equation for expectation values momentum, kinetic energy and hamiltonian as used in quantum mechanics (3 marks)
- b) Show that the momentum operator $-i\hbar \frac{\partial}{\partial x}$ is Hermitian operator. Obtain eigenfunction of momentum operator (17 marks)

QUESTION FOUR (20 MARKS)

- a) What are the two conditions for normalization of a wave function (2 marks)
- b) Consider a particle described by a wave function $\rho(r, t)$. Calculate the time-derivative $\frac{\partial \rho(r, t)}{\partial t}$ where $\rho(r, t)$ is the probability density and show that the continuity equation $\frac{\partial \rho(r, t)}{\partial t} + \nabla \cdot \vec{J}(r, t) = 0$ is valid, where $\vec{J}(r, t)$ is the probability current (18 marks)

QUESTION FIVE (20 MARKS)

- a) Derive Time – dependent Schrödinger equation and hence write it in three dimension given that the wave equation is $\Psi = Ae^{i(kx - \omega t)}$ (18 marks)
- b) The Schrödinger equation is a first order equation with respect to time discuss (2 marks)