



MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SPECIAL EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS

SMA 330: NUMERICAL ANALYSIS

DATE:

TIME:

INSTRUCTION TO CANDIDATES

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

- a) Convert the decimal number 81.5 to its binary form. (2 marks)
- b) Show that the operators μ and E commute. (3 marks)
- c) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)
- d) Prove that the forward difference of the quotient of two functions is given by
$$\Delta \left(\frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)}$$
(5 marks)
- e) Set up a Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)
- f) Find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4} (8 marks)

QUESTION TWO (20 MARKS)

- a) If E, μ and δ denote shift, average and central difference operators, in analysis of data with equal spacing h , prove the following

$$E^{1/2} = \mu + \frac{\delta}{2} \quad (2 \text{ marks})$$

- b) Solve the equation $x^3 = \sin x$. Considering various $\phi(x)$, discuss the convergence of the solution. (5 marks)
- c) For the following table of values, estimate $f(7.5)$, using Newton's backward difference interpolation formula (5 marks)
- e) Using the values given in the following table, find $\cos 0.28$ by linear interpolation and by quadratic interpolation and compare the results with the value 0.96106 (exact to 5D) (8 marks)

QUESTION THREE (20 MARKS)

- a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

- b) Find the positive solution of the transcendental equation $2\sin x = x$ (5 marks)
- c.) Find the cubic polynomial which takes the following values; $f(1)=24$, $f(3)=120$, $f(5)=336$, and $f(7)=720$. hence or otherwise, obtain the value of $f(8)$. (5 marks)
- d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Construct the forward difference table, where $f(x) = \frac{1}{x}$, $x = 1(0.2)2, 4D$ (4 marks)
- b) Find $\int_0^{10} \frac{1}{1+x^2} dx$ using Simpson's one third rule. (7 marks)
- c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FIVE (20 MARKS)

- a) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation $f(x) = 0$. (10 marks)

- b) Use the method in a.) to find the root of the equation $\tan x = x$ near $x = 4.5$ correct to four decimal places. (10 marks)